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SOCIETY OF MINING ENGINEERS of AIME

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Particle Breakage Studies In An Impact Crushing Environment

Research programs conducted by Allis-Chalmers Corporation are showing that the most efficient energy use in crushers is achieved when the forces applied to particles undergoing breakage are increased. Laboratory bench scale tests have been developed to confirm this phenomena, which was first observed in the field using *HYDROPHONE* crushers with automatic setting controls. A new testing machine is described and methods of maximizing such benefits are postulated.

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FOREWORD

Allis-Chalmers Corporation was founded in Milwaukee, Wisconsin, in 1847 as the E. P. Allis Company, which had an initial interest in manufacturing components in the grinding of flour. As such, its roots were started in reducing material size, which is covered by the generic term, comminution.

The Corporation evolved by a series of mergers into one of the world's first conglomerate companies.

Several key equipment suppliers to the mining and cement industries, in the late 19th century, became part of Allis-Chalmers and provided a broad base to build the modern technology we are using today.

Because this publication is recommending new approaches in solving old problems, it seems fitting to reflect on some past history to gauge the progress of our technological innovation. During this historical review it has been surprising to find features on equipment almost 100 years ago that one tends, from today's perspective, to think are relatively new. Some such developments had to be abandoned because of inadequate technology. The fact that such features existed at that time speaks volumes for the intelligence and inventive genius of the early crusher engineers. Some significant features will be mentioned in the following discussion.

The world's first truly successful compression type crusher was the Blake jaw crusher, first patented by E. W. Blake in 1858. The machine is still used today. Its greatest weaknesses were that it only crushed during half of the operating cycle, and it had only limited capability to crush fine.

Rotary vertical shaft crushers were under development by Gates Iron Works in Chicago in the 1860's, and following several aborted attempts the first successful gyratory type crusher was built, similar to the configuration shown in Fig. (A) according to patents issued to Charles M. Brown of Gates in 1878. It should be considered the forerunner of all gyratory and cone type crushers. The first unit was tested on loan to the Kirby & Howe Stone Co. of Iowa.

By January 8, 1881, Gates felt so confident of this new design that he advertised in the "Engineering & Mining Journal" of that date that he had "The Greatest Rock Breaker On Earth," and guaranteed that the crusher would do double the work of any upright convergent jaw crusher. He also openly challenged any competitor to a competitive trial.

By the turn of the century Gates claimed to have sold well over 4,000 crushers and was a clear leader in the field. They had also developed a spring relief protected machine to prevent problems associated with uncrushable tramp iron, but this was subsequently dropped from the model line.

Fraser and Chalmers, a company founded by two Scotsmen in Montana as a direct result of the Gold Rush around 1850, expanded their mining supply business by setting up in Chicago in 1860. They made Blake type jaw crushers and, ultimately, developed in 1890 the

C. M. BROWN.
Ore Crusher and Grinder.

No. 201,646.

Patented March 26, 1878.

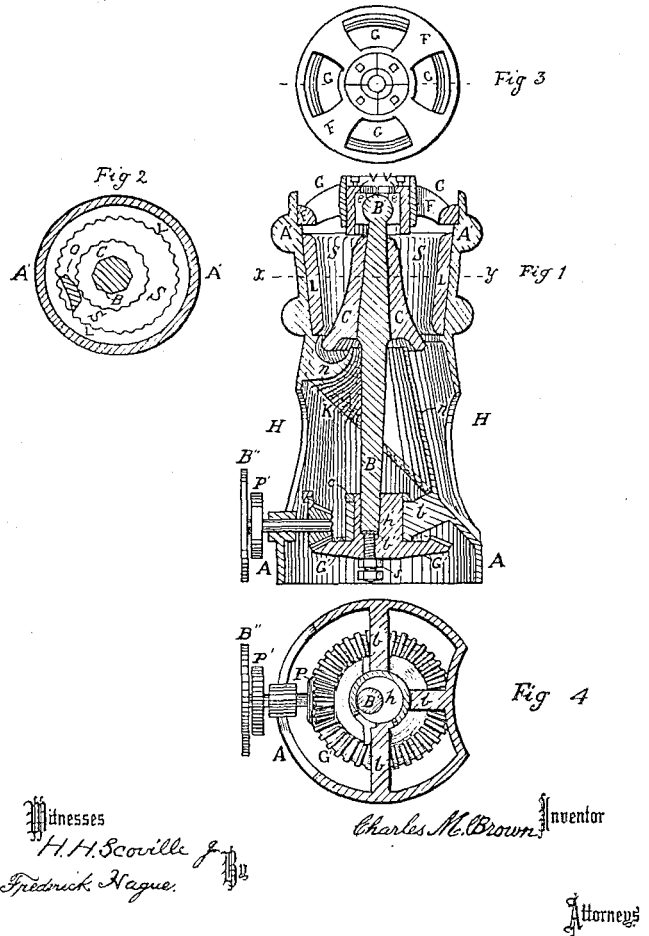


FIG. (A) C. M. BROWN GYRATORY CRUSHER
PATENT DRAWING

"COMET" crushing machine, which they claimed was the world's first adjustable under load gyratory unit.

In the latter part of the 19th century, Robert M. McCully of Philadelphia, Pa., produced the well-known gyratory crusher bearing his name. The machine had the first top suspended crushing shaft, all previous units having bottom shaft support, and great advantages were claimed for this arrangement. Power and Mining Machinery Works in Milwaukee made these machines under an exclusive license early this century.

All of these companies ultimately became part of the Allis-Chalmers conglomerate except the parts of Fraser and Chalmers outside the U.S.A., some of which still operate under that name.

The science of comminution was just getting started in the 1890's, so the rather brazen claims as to a machine's superior performance were at the mercy of aggressive marketing people of that era.

Apart from the already quoted advertisement of Gates, for posterity, it is worth quoting an excerpt from Fraser and Chalmers' "Comet Rock Crusher Catalog No. 6," which was published in 1897.

Read What It Will Do

The Comet Crusher is particularly adapted to crushing Railroad Ballast and Street Macadam, but we have designed it with sufficient strength to crush also with ease, Ores, Granite, Phosphate Rock, Corundum, Concrete, etc.

Our machines in many instances have been put to the severest possible tests and stood the work nobly.

The most wonderful capacity of this novel crusher is due to its large receiving capacity and easy motion. A car of rock can be dumped directly into the hopper, in fact men find it almost impossible to feed it. One of our No. C crushers working on exceedingly tough rock exceeded the combined efforts of three men to feed it, "fire flying all the while," as they said, and putting through eight yards of rock in twenty minutes, or say averaging sixteen yards every hour, including all delays, stoppages, etc.

The large Crusher E has put through more than a ton of rock every minute.

For the purposes for which it is designed this Crusher has no equal.

POWER & MINING MACHINERY CO.

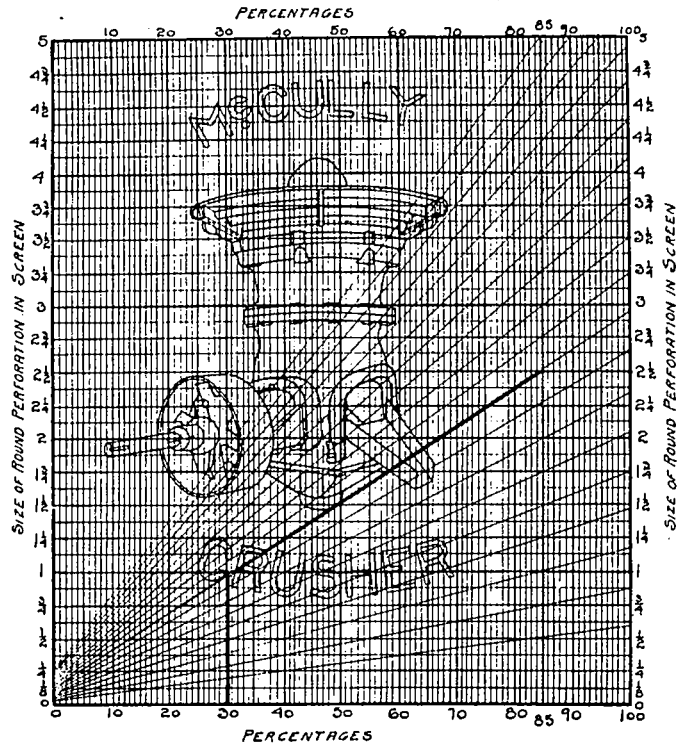


FIG. (B) McCULLY SIZE DISTRIBUTION GRAPH (1910)

Generally speaking, the manufacturer relied upon letters of testimonial to sell his equipment, and a generous number of these were normally included as part of any crusher catalog.

This same Fraser and Chalmers' Catalog No. 6 quotes a range of capacities for each size of various machines simply as "Capacity in tons per hour. Size of Macadam." No definition of particle size was given and naturally there could be little accountability.

The first attempts at defining power and capacity appear in Gates' Catalog No. 15, April 1900 edition, where a range of capacities produced through a 2½" ring are quoted for Style D ore breakers, which was qualified "according to the character of the rock."

A publication, undated, but probably printed about 1901, "McCully's Improved Rock and Ore Breaker," quotes capacities stated as being "tons of 2,000 lbs. Macadam or ballast size, 2½ inches" for a range of machines.

Probably one of the first attempts to relate crusher setting, size produced and power required appeared in Power and Mining Machinery Co.'s bulletin No. 4, published in 1910 for the McCully crushers. A graph used in such determinations is shown in Fig. (B).

It would appear that from this point up until the present, little has changed in our methodology concerning crusher application, although computers have been harnessed to speed up our calculating methods.

Application of crushing equipment for the most part has been a "Black Art."

In the old days, you needed a standard crusher to make coarse particles about 2½ inches (65 mm) and upwards, and a shorthead crusher if you wanted to crush finer. These terms were commonplace in manufacturer catalogs, such as McCully, between 1900 and 1920. Some manufacturers still use such terminology on machines designed since that period.

Clearly there have been substantial improvements to machinery including the application of automatic controls to improve productivity from a crusher. Our knowledge of the way a material behaves under processing conditions has improved immensely.

It seems inappropriate that we still use, basically unchanged, the same 70-year-old application methods for most crushing calculations.

This publication suggests a more fundamentally based method of applying crushing machines in terms of scientifically based energy and size parameters. Hopefully it will contribute to the better understanding of these processes.

Our past experiences are the basis of determining the need for such improved technology.

ABSTRACT

Research programs conducted by Allis-Chalmers Corporation are showing that the most efficient energy use in crushers is achieved when the forces applied to particles undergoing breakage are increased. Laboratory bench scale tests have been developed to confirm this phenomena, which was first observed in the field using *HYDROCONE* crushers with automatic setting controls. A new testing machine is described and methods of maximizing such benefits are postulated.

INTRODUCTION

Rapidly increasing operating costs for minerals beneficiating plants continue to be the biggest single problem in maximizing profitability from these operations. The average world inflation rate has been increasing over the last decade and shows little sign of easing. The threat of continued increases in the price of fuel oil (some analysts predict a 25% increase from \$32 to \$40 per barrel in 1981) will eventually increase the cost of electrical power, in direct proportion for most users. This will undoubtedly cause closure of some lower grade ore bodies unless energy utilization efficiencies, particularly in comminution, can be improved.

Most of the recent literature concerning comminution performance improvement has been directed at grinding mill performance. It can be expected that more refined control systems will improve the overall milling energy efficiency, which is normally the largest single cost component of production. However, published gains by such methods to date appear to be limited to something less than 10%.

The second largest cost for comminution processes is normally that for wear metal consumed in grinding operations.

Allis-Chalmers has continuing research programs into all forms of comminution processes involving crushing and grinding. Improved crushing technology shows the way to reducing both energy and wear metal consumption mainly by producing finer feed which will improve downstream grinding mill performance.

Because there has been less emphasis on crushing technology than milling, this paper will also discuss objectively the comparisons of energy and wear metal for the two processes.

A new testing procedure for studying crushing phenomena, presently being perfected by Allis-Chalmers, is described for the first time. These bench scale laboratory tests will give more accurate prediction of both energy requirements and size distribution produced in commercial crushing processes. As a direct result, this machine will allow more accurate comparisons to be made in capital and operating cost expenditures for various combinations of crushing and milling processes.

These new testing procedures can be run on small samples including pieces of drill core material. They

could be part of testing and feasibility studies for most new concentrators. The same methods can be used to determine likely yield of various sized crushed products and, therefore, benefit crushed stone producers.

BACKGROUND

The theoretical and practical phenomena concerning comminution processes have received considerable attention in the literature and are not discussed here in any detail. Instead, the breakage studies in this paper are based on an empirical treatment of the fundamental relationships between energy and the size distributions of processed particles that have been observed both in the laboratory and in large-scale, commercial cone-crushing operations.

Because of the bewildering number of variables encountered when studying comminution processes, most investigators have preferred to assume that the size distribution generated in milling and crushing processes bears some relatively fixed relationship such as those described by Gates-Gaudin-Schuhmann¹ or Rosin-Rammler.²

Fred Bond, in his "Third Theory of Comminution,"³ used the former, essentially assuming that size versus cumulative percent passing that size was represented by a straight line of assumed slope 0.5 below the 80% passing size. Based on this assumption, Bond derived his well-known relationship:

$$W = \frac{10 W_i}{\sqrt{P_{80}}} - \frac{10 W_i}{\sqrt{F_{80}}} \dots\dots\dots (1)$$

where W = work in KWH/short ton
 W_i = Work Index
 F_{80} = 80 percent passing size of the feed (micrometers)
 P_{80} = 80 percent passing size of the product (micrometers)

The Work Index for rod and ball mills can be determined from laboratory tests and, as demonstrated by Rowland,⁴ the relationship gives us a reasonably accurate tool for the design of rotary grinding mill circuits.

Bond's methods have been less successful in predicting fine crushing performance, however, primarily because the typical crusher feed and product distributions do not meet the assumed conditions necessary for the satisfactory application of his equation (see Fig. (1)).

It is most evident that the curved lines appearing on Fig. (1) do not represent a Gates-Gaudin-Schuhmann size distribution. It is therefore not surprising that Bond's procedures do not work well in this situation. The Rosin-Rammler distribution has also been found inadequate to generally describe crusher products.

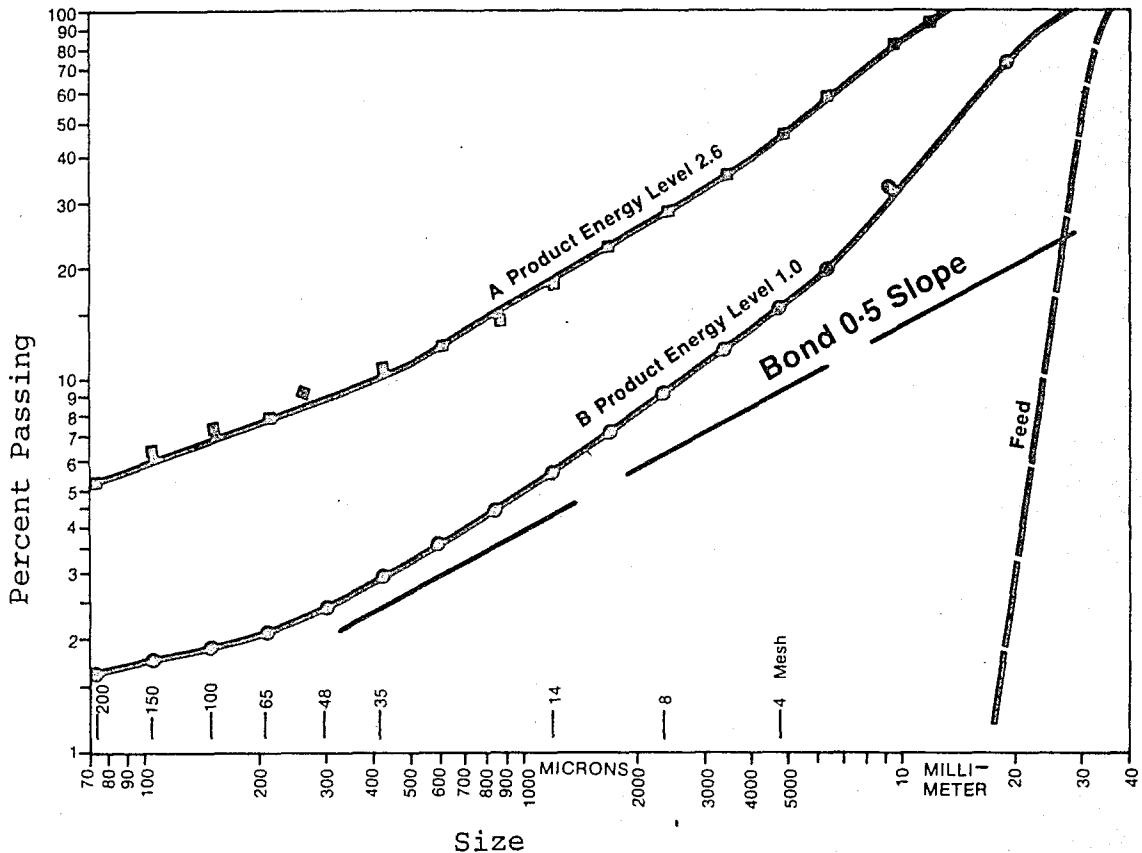


FIG. (1) TYPICAL CONE CRUSHER FEED AND PRODUCT SIZE DISTRIBUTIONS

Work during the early 60's led to the concept of comminution as a repetitive process, with each step consisting of two basic operations — the selection of a particle for breakage and the subsequent breakage of this particle by the machine. In this approach, the process under investigation is modelled by combining the particle selection/breakage event with information on material flow in and out of the comminution device.

Most workers who have used this approach⁵ have considered size reduction to be the result of the mechanical operation of the comminution device. This mechanical operation consumes the energy, and size reduction is merely a result of this energy consumption. This viewpoint is reasonably valid for tumbling mills where energy input tends to be constant and the proportion of the energy that is usefully consumed in particle breakage is low (<10%). It does not appear to be valid in compression crushers, however, since breakage energy is a significant proportion (>50%) of the total energy input to the crusher and markedly different power rates (energy input per unit of crusher feed) can be obtained by varying ore feedrates and/or crusher parameters such as closed side setting. It will therefore be necessary to include energy information in any model of the crushing process before it will be possible to accurately predict crusher performance. The inclusion of this energy-size information will significantly increase the complexity of these models.

The single-particle breakage event has been the subject of several studies.^{6, 7, 8} Most of these have utilized only sufficient energy to break the particle and do not

simulate commercial crushing operations where energy levels are such that catastrophic repetitive breakage usually takes place. This approach to the study of comminution processes does yield valuable information, however, and it is unfortunate that it has not received greater attention.

TEST PROCEDURES FOR THE CHARACTERIZATION OF CRUSHER PERFORMANCE

The Bond Impact Work Index method has been an industry standard for the determination of crusher power requirements but was originally developed⁹ to ensure that sufficient power was connected to primary gyratory crushers. In this method, pieces of rock are fractured by trial and error in the test device shown in Fig. (2), until sufficient impact energy has been applied to break the rock.

Normally, the rock breaks in halves, and in most tests only two and seldom more than three large pieces are observed after fracture. No size distribution information is used in calculating the Bond Impact Work Index from the formula:

$$WI = 2.59 \frac{\text{Average Impact in ft-lbs/inch}}{\text{Specific Gravity}} \quad .(2)$$

This mode of breakage is similar to that occurring in a primary gyratory crusher where power rates (energy input per tonne of feed) are low (approximately 0.1

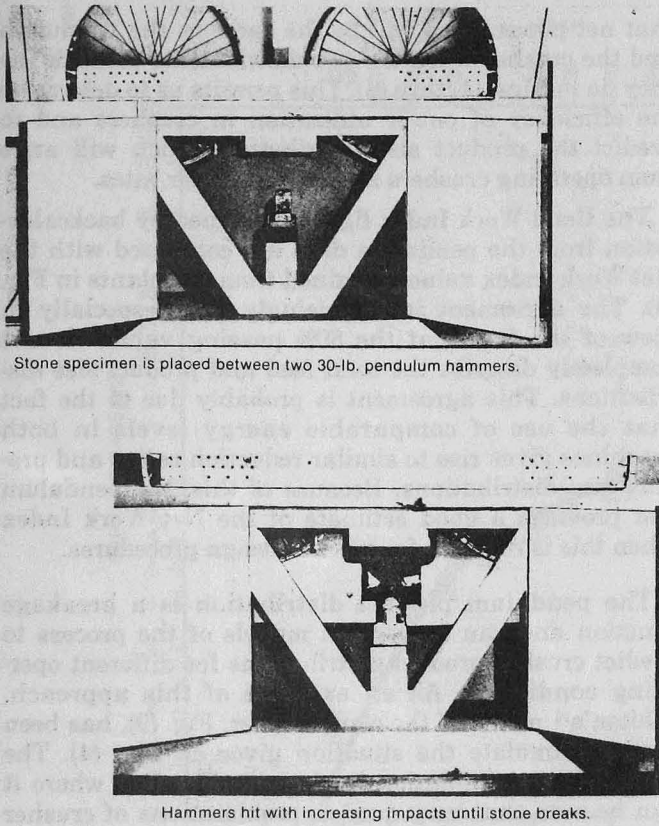


FIG. (2) BOND IMPACT WORK INDEX TEST DEVICE

KWH/tonne). The procedure works quite well for this type of crusher but tends to understate power requirements in fine crushers where power rates are typically much higher (upwards from 0.25 KWH/tonne).

Because of this, a research program was instituted by Allis-Chalmers Comminution Task Force Committee to break rock in a manner more analogous to that observed within commercial fine crushers. A pendulum type test device similar in most respects to that developed by the United States Bureau of Mines¹⁰ and shown diagrammatically in Fig. (3), was built and has been used in an extensive test program to determine whether it would be possible to predict cone crusher performance.

The rock samples selected for crushing in this device are usually minus 38mm (1-1/2"), plus 19mm (3/4") in size. The sample rock is weighed and then placed between the platens. The end of the rebound platen is placed in contact with the rebound pendulum and the crushing pendulum is raised to a predetermined vertical height which depends on the size of the sample. The crushing pendulum is then released — after striking the crushing platen and breaking the rock, the remaining energy is transferred via the rebound platen to the rebound pendulum. The horizontal distance that the rebound pendulum travels is recorded by displacement of a marker and is subsequently converted to a vertical height.

The energy used in crushing the sample can be approximated from an energy balance of the system:

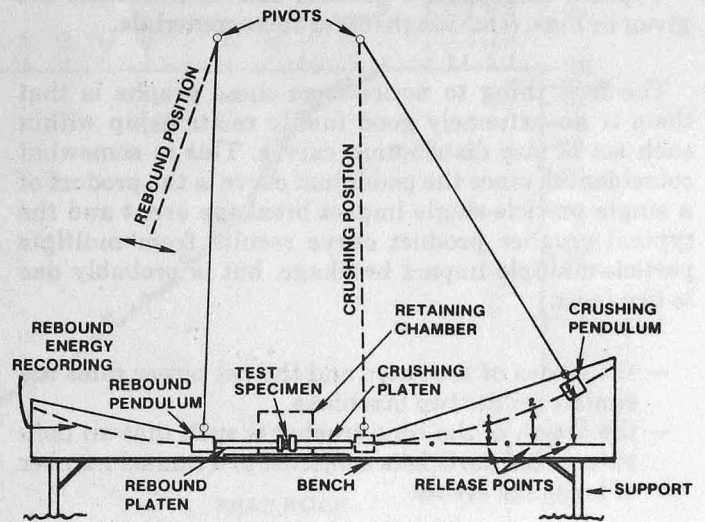


FIG. (3) PENDULUM CRUSHING DEVICE DIAGRAMATIC NO SCALE

$$E_c = E_1 - KE - E_2 - E_L \text{ (Kg-mtrs)} \dots\dots (3)$$

- where E_c = crushing energy
- E_1 = crushing pendulum potential energy (before release)
- KE = kinetic energy of the two platens
- E_2 = rebound pendulum maximum potential energy (after crushing)
- E_L = system energy loss (sound, heat, vibration)

The system energy loss, E_L , is determined by plotting E_L as a function of the initial height of the crushing pendulum with no rock present. The major portion of this loss is by vibration. It is felt that the difference between system energy losses with and without rock present in the system is minimal as long as enough initial energy is supplied to result in a small elevation of the rebound pendulum.

The fragments from several rock samples broken under identical conditions were combined for each of the size analyses reported in this paper. Bond Work Indices were also backcalculated from the data using the standard formula, ie.

$$WI = \frac{(E_c/\text{particle weight})_{av}}{\sqrt{P_{80}}} - \frac{10}{\sqrt{F_{80}}} \dots\dots (4)$$

where F_{80} is defined as the length of the side of the cube of the same weight as the average particle.

Confirmation of the ability of the procedure to provide information suitable for the prediction of crusher performance was obtained by taking feed samples from 31 commercial operations treating a wide range of rocks and ores. At the time of taking a feed sample for laboratory testing in the pendulum device, relevant performance data such as power, feed rate and size distributions for feed and product were taken on the operating crusher. Several thousand rocks have been broken during tests with the device over the past 3 years.

Typical, comparative product size distributions are given in Figs. (4) through (6) for three materials.

The first thing to notice from these graphs is that there is an extremely good family relationship within each set of size distribution curves. This is somewhat coincidental, since the pendulum curve is the product of a single particle-single impact breakage event and the typical crusher product curve results from multiple particle-multiple impact breakage, but is probably due to two facts:

- the modes of breakage and the net power rates are similar for the two machines
- the action of the cone crusher is such that an individual feed particle is subjected to a limited number of breakage events

In order to show that the pendulum product size distribution is sensitive to power rate, several tests have been run on the same feed material at different levels of pendulum input energy. Typical results are shown in Fig. (7) as Schuhmann size distribution (log-log) plots. It can be seen that increasing amounts of fine material are produced with increasing energy input. The same effect was previously demonstrated for an operating crusher in Fig. (1). We can, therefore, conclude from this

that net power rates will be the same in the pendulum and the crusher when the two distributions coincide (as they do in Figs. (4) thru (6)). This permits us to determine the efficiency of power utilization in crushers and to predict the product size distribution which will arise from operating crushers at different power rates.

The Bond Work Index figures obtained by backcalculation from the pendulum data are compared with the Net Work Index values obtained from the plants in Fig. (8). The agreement is surprisingly good especially in view of the fact that the 80% passing values do not completely describe the total feed and product size distributions. This agreement is probably due to the fact that the use of comparable energy levels in both machines gives rise to similar reduction ratios and product size distributions. Because of this, the pendulum test provides a good estimate of the Net Work Index when this is required for current design procedures.

The pendulum product distribution is a breakage function and can be used in models of the process to predict crusher product distributions for different operating conditions. As an example of this approach, Whiten's¹¹ model of the cone crusher, Fig. (9), has been used to simulate the situation given in Fig. (4). The result of this simulation is given in Fig. (10) where it can be seen that very good approximations of crusher performance can be obtained.

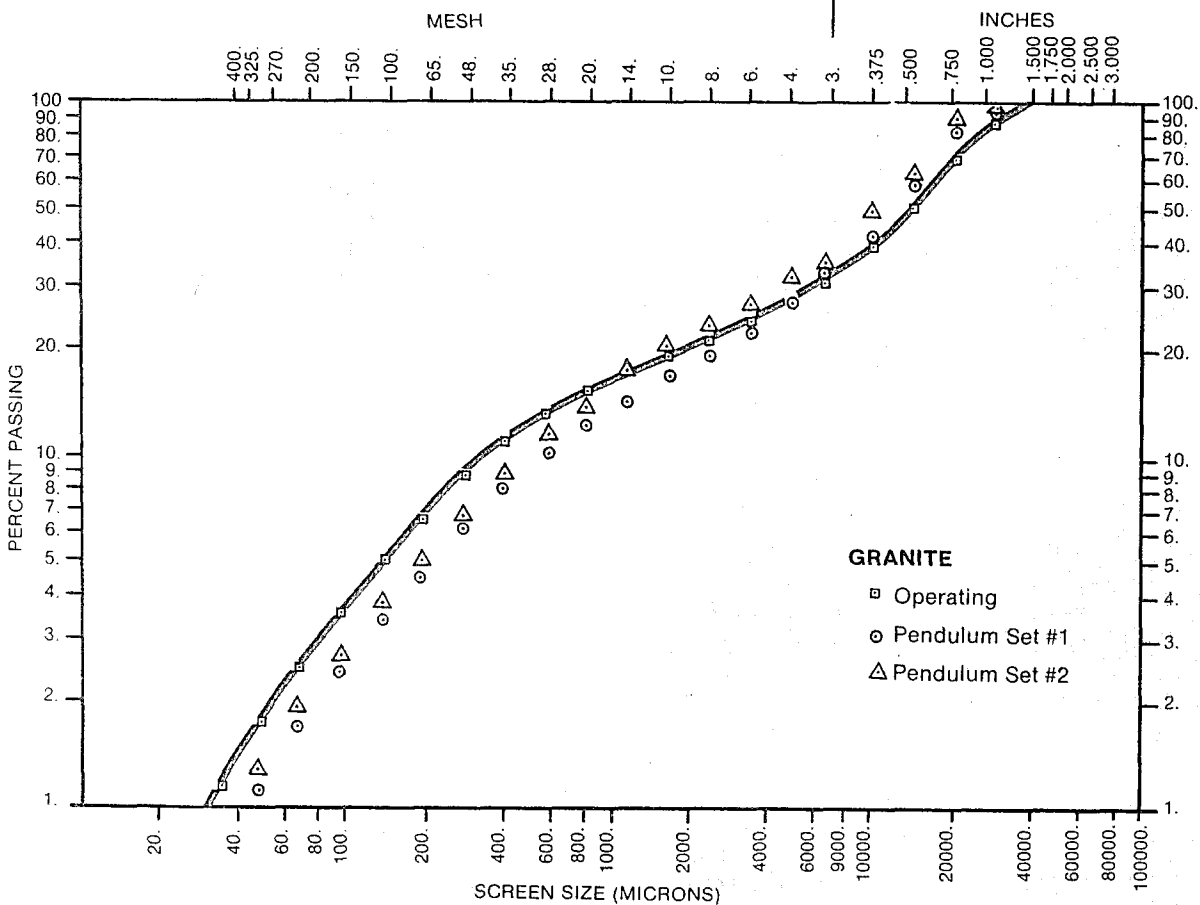


FIG. (4) COMPARATIVE PRODUCT SIZE DISTRIBUTIONS — GRANITE

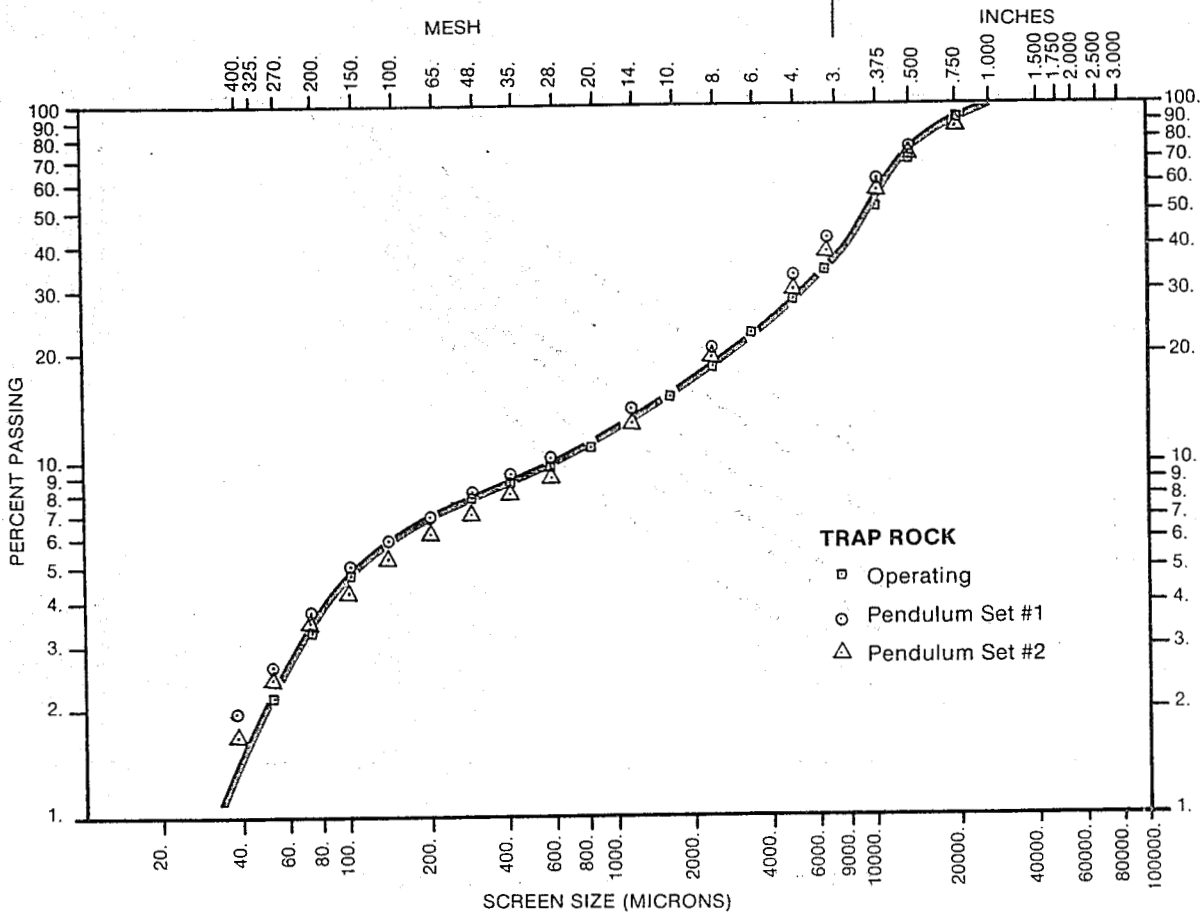


FIG. (5) COMPARATIVE PRODUCT SIZE DISTRIBUTIONS — TRAP ROCK

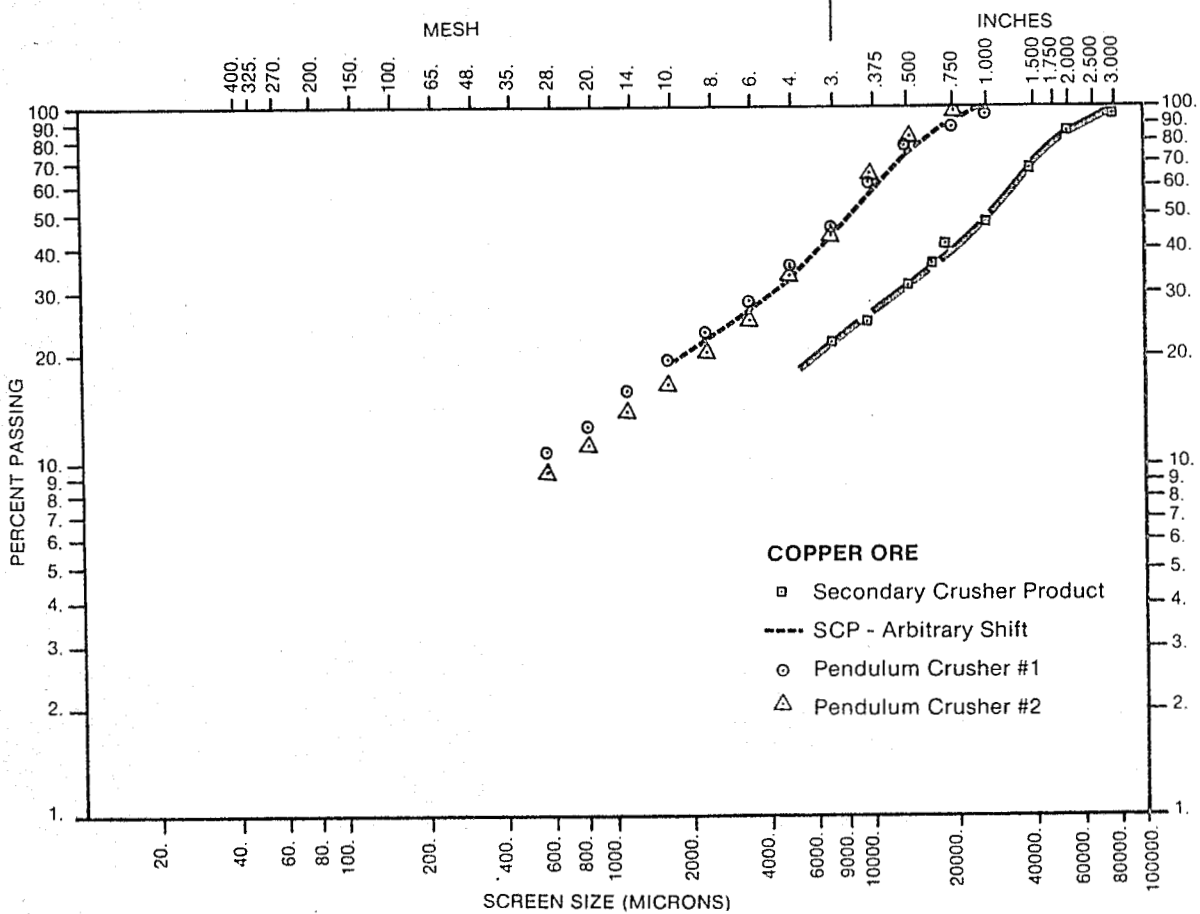


FIG. (6) COMPARATIVE PRODUCT SIZE DISTRIBUTIONS — COPPER ORE

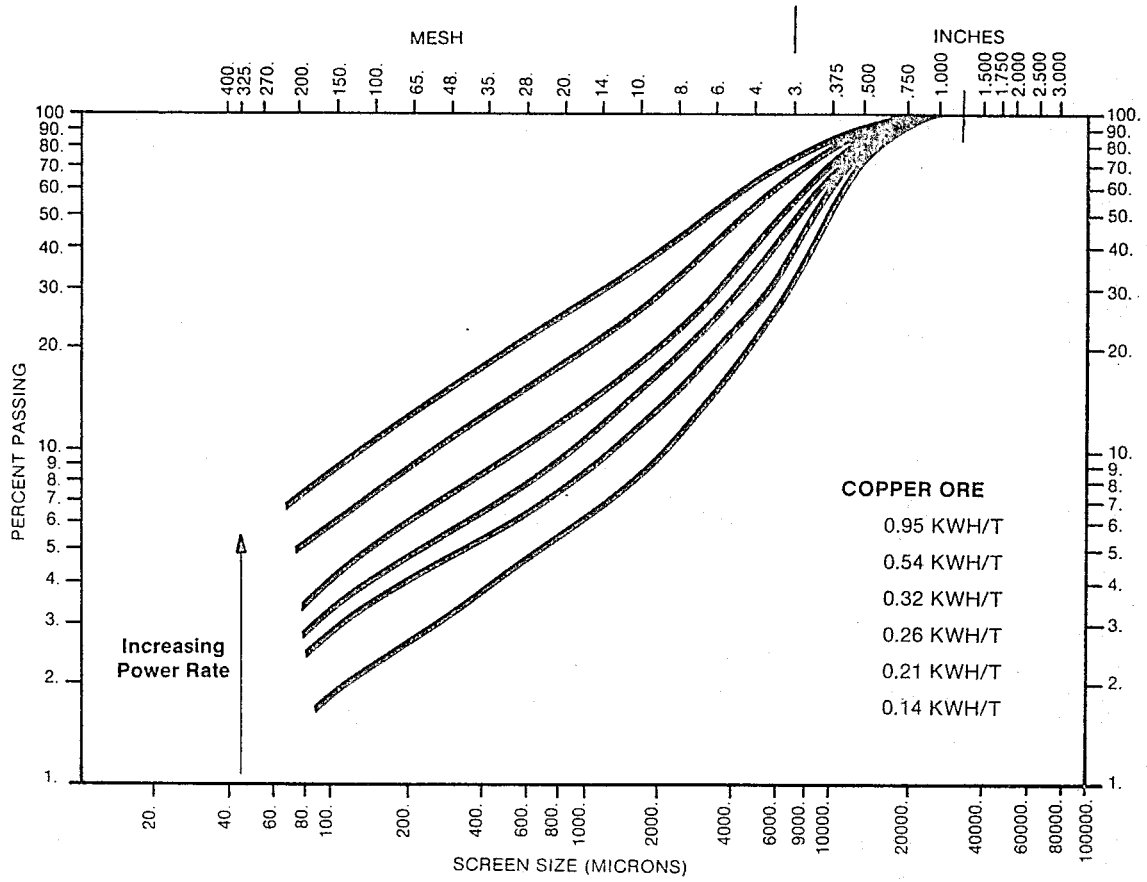


FIG. (7) EFFECT OF INCREASING POWER RATE — COPPER ORE

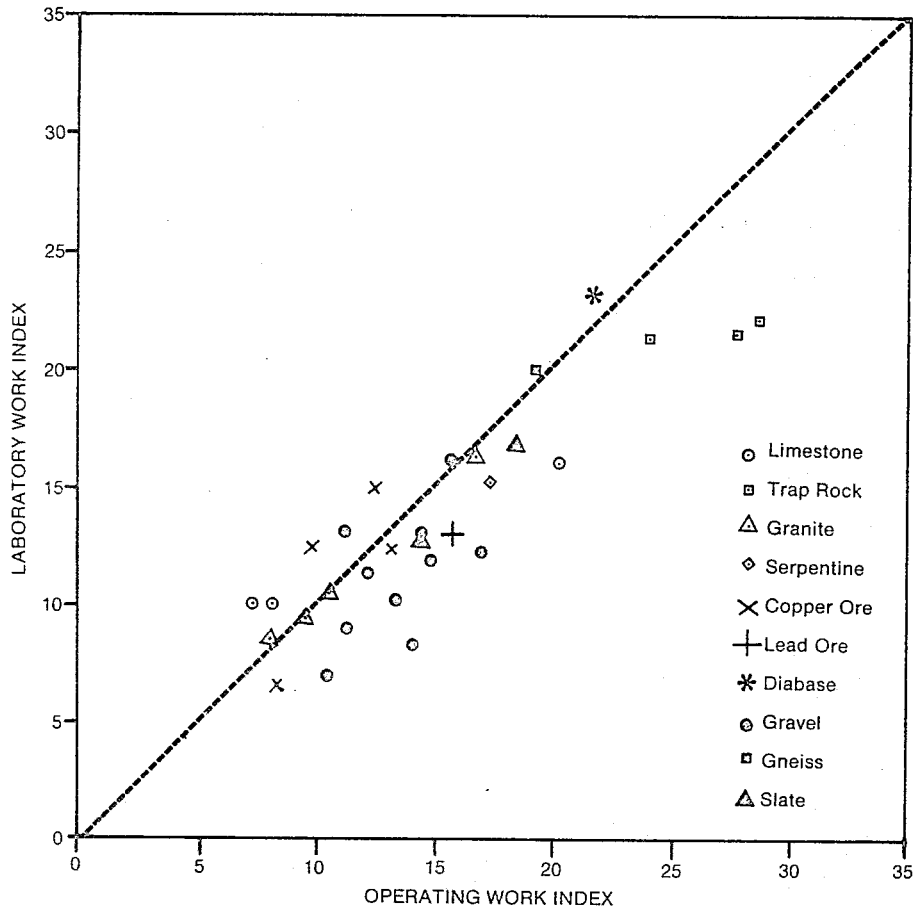
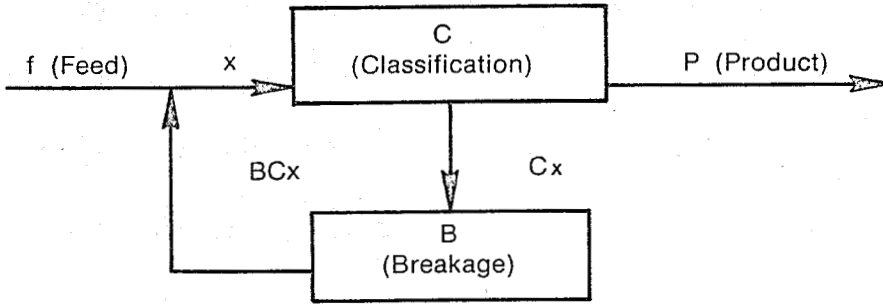


FIG. (8) COMPARISON OF PLANT AND PENDULUM WORK INDICIES



Mass Balances at the Nodes Give the Equations

$$f + BCx = x$$

$$x = Cx + P$$

Eliminating x and Rearranging Gives

$$P = (I - C) (I - BC)^{-1} f$$

f, p and x are Vectors Giving the Mass Flow Rates in Each Size Fraction

B and C are the Breakage and Classification Matrices, I is the Unit Matrix

FIG. (9) WHITEN'S MODEL OF A CONE CRUSHER¹¹

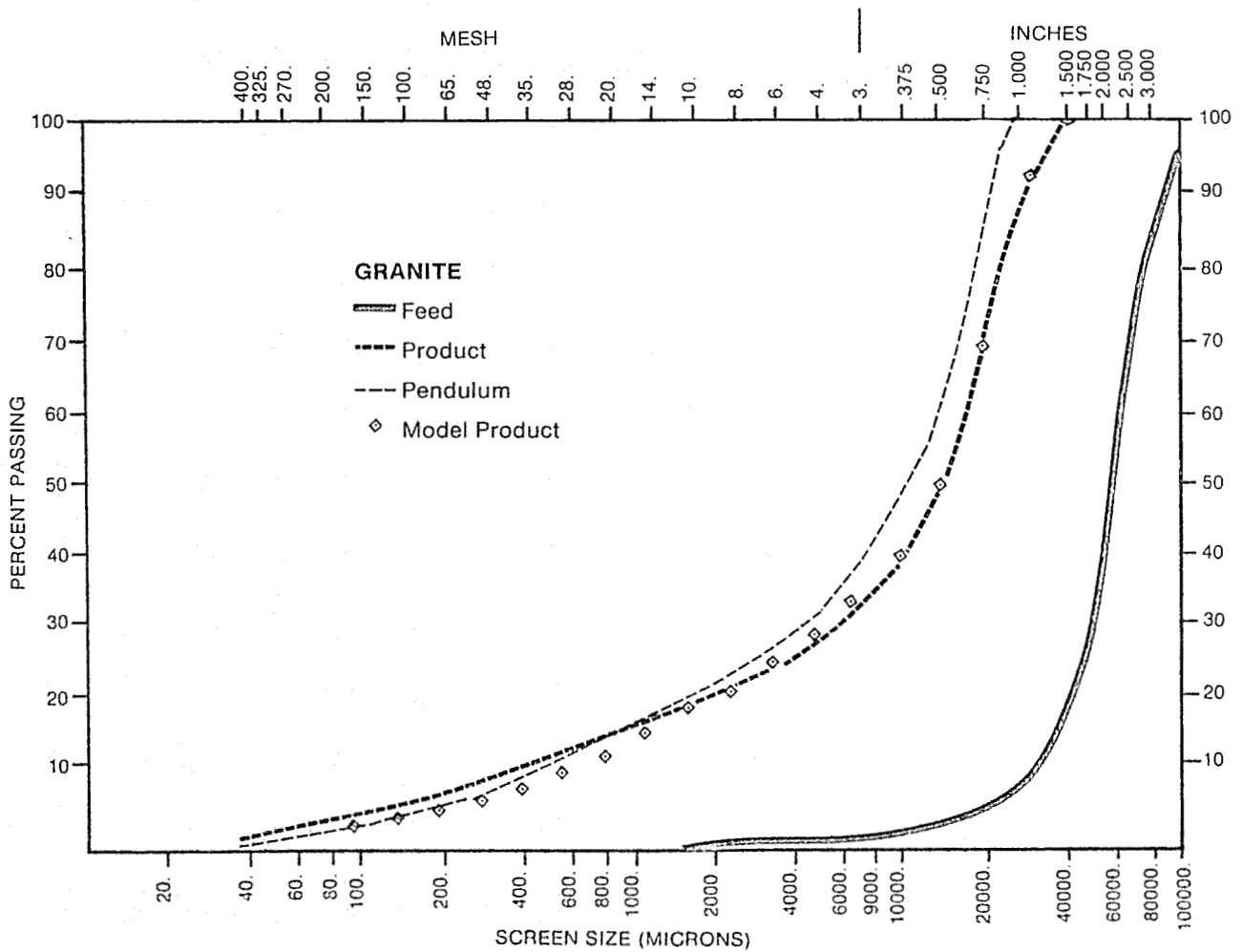


FIG. (10) CONE CRUSHER SIMULATION — COMPARATIVE SIZE DISTRIBUTIONS FOR GRANITE

The selection (or classification) function used in this simulation was determined by backcalculation. Work is presently underway to quantify this function.

The writers are firmly of the opinion that results to date prove that the use of this pendulum device can give more energy-size reduction information in a form readily useable for crusher application. The data can be generated in less time and from a much smaller sample than is required for pilot plant testing. Our present pendulum tester is a research tool and is currently being modified for use in commercial testing of minerals and rocks. More details of this device will be given at a later date.

APPLICATION OF TECHNOLOGY

From the discussion so far it is obvious that two energy factors determine a crusher's productivity:

- (1) Total power drawn affects the quantity produced of any given size or range of sizes.
- (2) The energy applied, previously defined as the crusher's Power Rate (kilowatt hours per ton of

feed) will determine the size reduction. This has just been clearly demonstrated by the pendulum tests.

In a previous paper¹², we have shown that the theoretical forces achieved in a crushing chamber are a direct result of the design geometry and driving power applied to the crusher. Machines from three manufacturers can theoretically apply different crushing forces to processed material, according to Table (1).

The energy intensity within the crushing chamber will differ for various machines. The wide range of eccentric throws and connected powers for various units will give different crushing impacts, which might be considered analogous to the pendulum. The angle of the mantle for the crushing head varies between manufacturers as does the length of the crushing zone from less than 200mm (8 inches) to more than 600mm (24 inches) but as has been indicated from the pendulum size distribution correlations it is the energy input to the feed material which determines the reduction and size distribution. Eccentric throw and chamber shape appear to have little influence.

TABLE (1)
THEORETICAL UNRESOLVED CRUSHING FORCE

| From Formula: $F = \frac{\text{Torque}}{\text{Half Eccentric Throw}} = \frac{63025 \times \text{HP}}{S \times \text{ECC Throw}} \text{ (British Units)}$ | | | | | | | | |
|--|--------|--------|--------|--------|---------|--------|--------|--|
| | | | 2 | | | | | |
| "A" Manufacturer Model* | 36 | 45 | 51 | 60 | 84 | | | |
| Kilowatts | 93 | 149 | 149 | 224 | 373 | | | |
| Horsepower | 125 | 200 | 200 | 300 | 500 | | | |
| S Gyr/Min | 350 | 290 | 290 | 260 | 210 | | | |
| Throw (in.) | 1.13 | 1.38 | 1.38 | 1.5 | 2.0 | | | |
| mm | 29 | 35 | 35 | 38 | 51 | | | |
| F Force Pound | 40,015 | 63,222 | 63,222 | 96,961 | 150,000 | | | |
| kg | 18,156 | 28,685 | 28,685 | 43,993 | 68,058 | | | |
| "B" Manufacturer Model* | 3' | 3' | 4' | 4-1/4' | 5-1/2' | 5-1/2' | 7' | |
| Kilowatts | 45 | 56 | 75 | 112 | 149 | 149 | 224 | |
| Horsepower | 60 | 75 | 100 | 150 | 200 | 200 | 300 | |
| S Gyr/Min | 385 | 385 | 240 | 240 | 237 | 237 | 212 | |
| Throw (in.) | 2.13 | 2.31 | 2.69 | 3.0 | 3.5 | 3.88 | 4.0 | |
| mm | 54 | 59 | 68 | 76 | 89 | 99 | 102 | |
| F Force Pound | 9,244 | 10,620 | 19,538 | 26,260 | 30,391 | 27,415 | 44,593 | |
| kg | 4,149 | 4,819 | 8,865 | 11,915 | 13,789 | 12,439 | 20,233 | |
| "C" Manufacturer Model* | 36 | 48 | 66 | 1100 | 1500 | 1900 | | |
| Kilowatts | 75 | 149 | 224 | 112 | 182 | 298 | | |
| Horsepower | 100 | 200 | 300 | 150 | 250 | 400 | | |
| S Gyr/Min | 307 | 268 | 220 | 303 | 267 | 219 | | |
| Throw (in.) | 2.0 | 2.6 | 2.75 | 2.2 | 2.75 | 3.5 | | |
| mm | 51 | 66 | 70 | 56 | 70 | 89 | | |
| F Force Pound | 20,529 | 37,627 | 62,504 | 28,364 | 42,813 | 65,780 | | |
| kg | 9,314 | 17,072 | 28,359 | 12,869 | 19,425 | 29,846 | | |

*NOTE: Some manufacturers do not give all this specification information in sales literature, and many of these measurements here were made in the field on operating equipment. Even if correct in one instance, the specifications could be different for different machines. For accurate data, the person studying such information should refer to the crusher manufacturer.

This information is only published so the reader can get an order of magnitude comparison for setting up his own technical study.

To make the finest product, we should be running the crusher with the lowest practical feed rate and applying the maximum crushing force. This will give the highest power rate but there are physical limits to operating in this mode, especially if the crusher is run at fixed setting. To make a coarse product the power rate should be no greater than that needed to generate that size. The power drawn in any case should be maximized to yield the greatest quantity of the required product.

There has been adequate reporting by people such as Szaj¹³ and Lynch¹⁴ that operating crushers at fixed setting most often demands that the average operating power be limited to only 50 or 60% of that connected, to guard against crusher stalling and minimize mechanical damage. Control devices such as those by Lindgren *et al*¹⁵ have been developed to maximize power drawn and productivity through fixed setting crushers.

The power drawn by a crusher can be changed by varying the feed rate and/or the setting, as shown in Figs. (11) and (12). A small change in setting can bring about a large change in power rate, Fig. (13). Changing feed rate near optimum power draw gives small change to crusher power rate and the reduction ratio remains approximately the same.

The rapid fluctuation in crusher power draw that is normally associated with high power rate operation

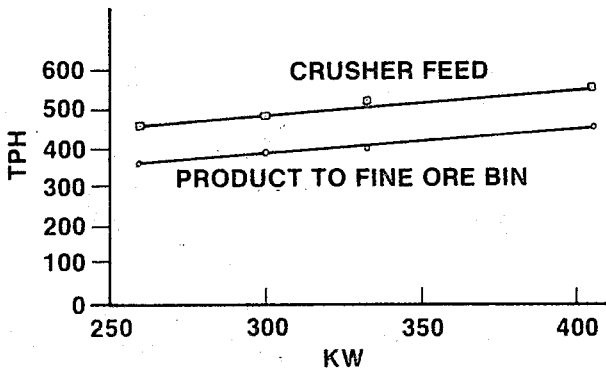


FIG. (11) FIXED SETTING FEED RATE POWER RELATIONSHIP

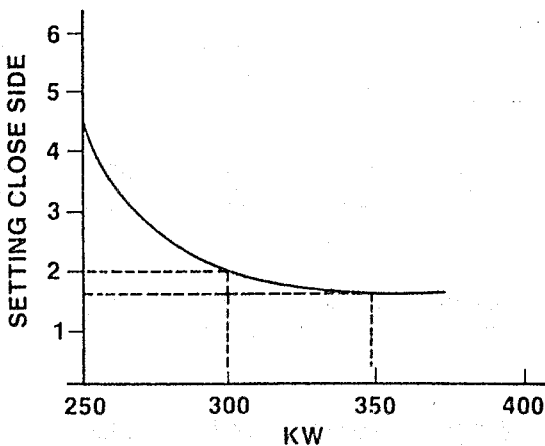
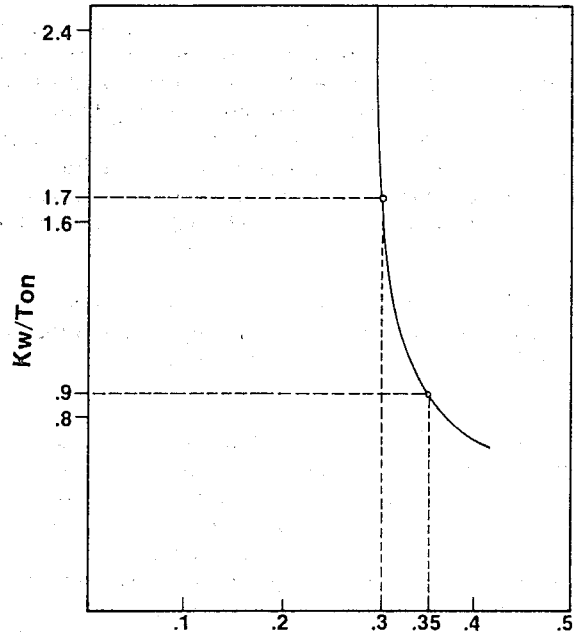


FIG. (12) VARIABLE SETTING POWER RELATIONSHIP



CLOSED SIDE SETTING

FIG. (13) SMALL CHANGE IN SETTING GIVES LARGE CHANGE IN CRUSHERS POWER RATE

makes it desirable to fit a crusher with an automatic setting control system as shown in Fig. 14 to guard against mechanical overloading. At the same time this maximizes the power rate and average power drawn for a particular configuration.

It has been determined that within a cone crushing chamber, machines using automatic setting regulation can normally operate at more than twice the power rate compared to units operating in a fixed setting mode.

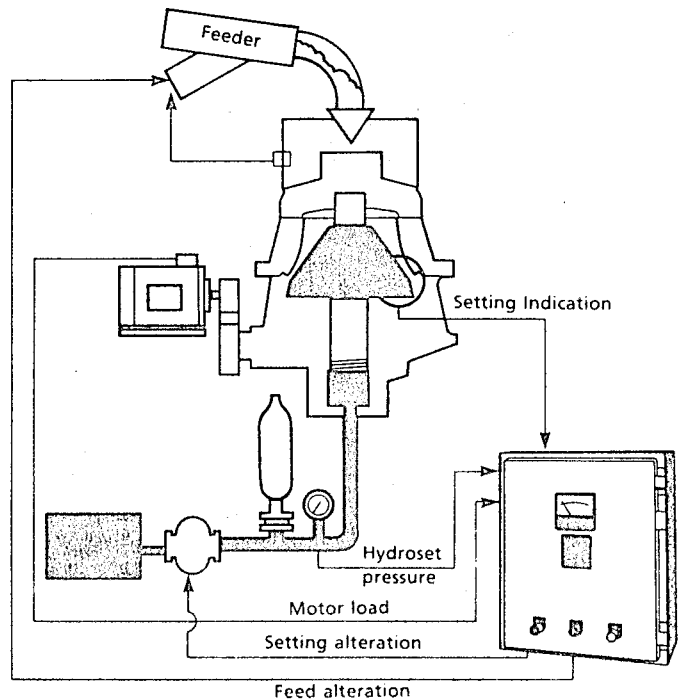


FIG. (14) ALLIS-CHALMERS HYDROCONE CRUSHER WITH AUTOMATIC SETTING REGULATION AND FEED RATE CONTROL

The device in Fig. (14) works by constantly trying to reduce the machine's setting according to a program, thereby maximizing power drawn within preset limits. If the crusher overloads, it opens up according to another program. Feed rate can be another variable controlled within the same system. The combined setting/feed rate system can be appropriately coordinated to give a crusher power rate control, thus effectively determining reduction ratio as will be seen in more detail following.

It has been demonstrated that both capacity and quantities of finer sizes produced, Fig. (15), can be improved substantially by using automatic setting controls in a cone crusher.

It can be seen that the yield of minus 13 mm material for the automated set crusher is approximately twice that for the fixed set machine. Also notice that the percent minus 13 mm and 9.5 mm in the crushers' discharge is about 10% higher even though the crushers are operating at close to the same close side setting. One of the main reasons for the differential in this test is the larger eccentric throw in the fixed set unit. The smaller eccentric throw and automatically controlled setting increased power draw to give a much higher crushing force. This is analogous to a large pendulum energy in the laboratory tests.

By adjusting both the feed rate and power together, we can change the power rate and, hence, reduction, as is shown from a field test result, Fig. (16). The automated setting and power drawn was almost the same for both operating conditions. The large difference in reduction was due to the different power rates. Superimposed pendulum results again demonstrate the family curve relationships for the laboratory tests.

The operating conditions for the crushers are diagrammatically shown in Figs. (17) and (18).

It is clearly shown that the same energy applied to more material results in a much coarser product. In these tests the percent minus 12.7 mm (1/2 inch) changed from approximately 95% to 65% passing when the feed rate was doubled (power rate was effectively reduced by half). The automated setting control adjusted the crusher to the same power draw setpoint (320 KW). Because there is an exponential relationship between power drawn and close side setting, there was only a small 1 mm change between the tests. If an application engineer had been using a catalog, he might have computed the size distribution as 60% passing the setting. Clearly there would be a considerable error if this happened.

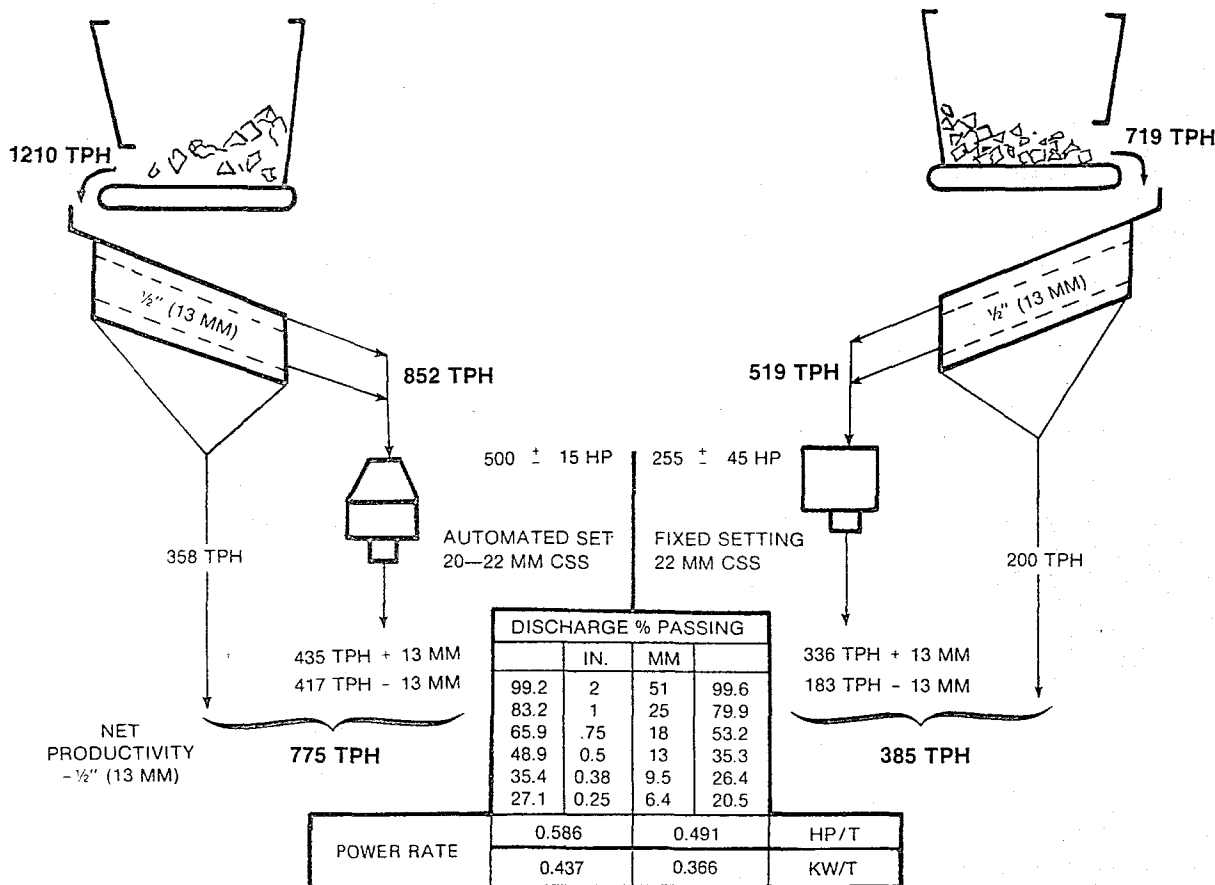


FIG. (15) TWO 84" (2134 MM) DIA. MANTLE SECONDARY CONE CRUSHERS TEST COMPARISON

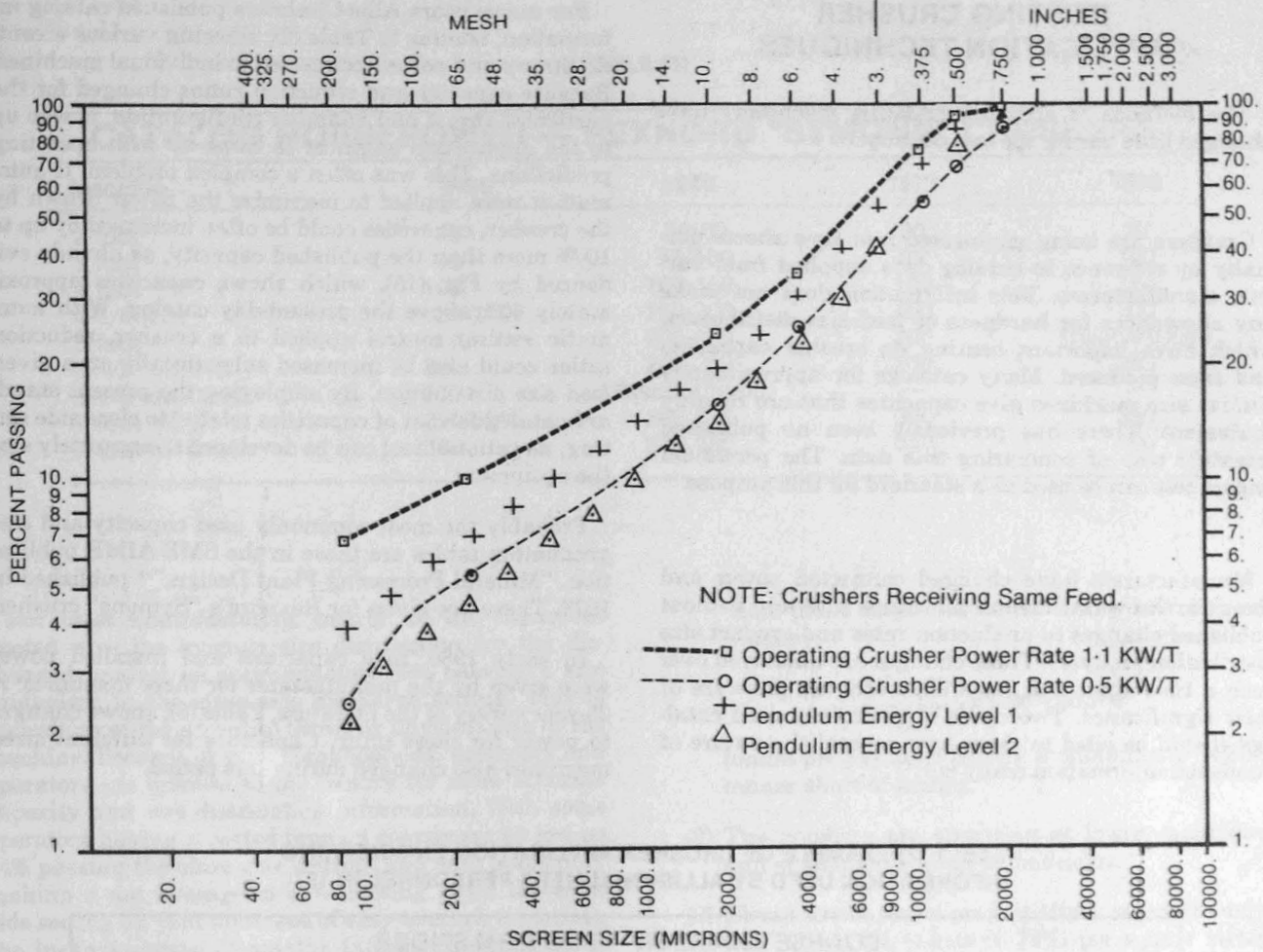


FIG. (16) OPERATING POWER RATES AND PENDULUM PRODUCT SIZE DISTRIBUTIONS

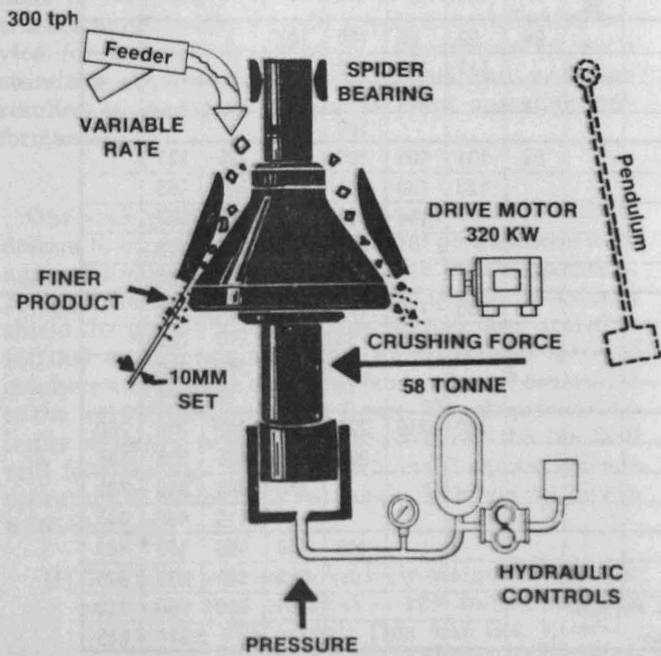


FIG. (17) HIGH POWER RATE CRUSHING (1.1 KW/T) ADJUSTING BOTH FEED RATE AND CRUSHER SETTING (MORE ENERGY PER ROCK CRUSHED)

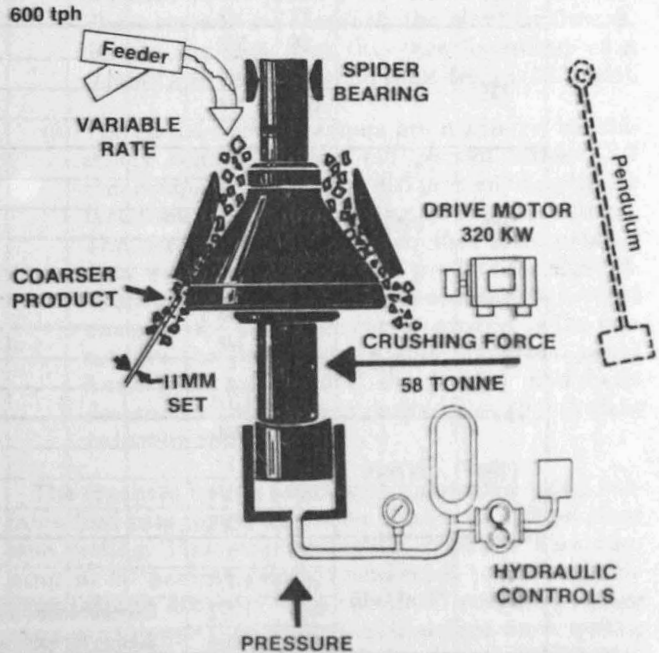


FIG. (18) LOW POWER RATE CRUSHING (0.53 KW/T) ADJUSTING BOTH FEED RATE AND CRUSHER SETTING (LESS ENERGY PER ROCK CRUSHED)

EXISTING CRUSHER APPLICATION TECHNIQUES

The methods of applying crushing machinery have changed little during the last century.

Crushers are being engineered into flow sheets normally by reference to catalog data supplied from various manufacturers. This information does not make any allowances for hardness or feed size distribution, which have important bearing on crusher capacities and sizes produced. Many catalogs for approximately similar size machines give capacities that are roughly equivalent. There has previously been no published scientific way of comparing this data. The pendulum impact test can be used as a standard for this purpose.

Manufacturers have changed connected power and eccentric throws on various machines with and without published changes to production rates and product size distribution analyses. These changes have occurred over such a time span that most engineers are unaware of their significance. Two cases of commonly used catalogs should be cited to show how general the nature of published information really is.

For many years Allis-Chalmers published catalog information, similar to Table (2), showing various eccentric throws and powers connected to individual machines. Because capacity and reduction ratios changed for the particular throw and chamber configuration, it was up to the application engineer to come up with operating predictions. This was often a complex problem. If automation were applied to maximize the power drawn by the crusher, capacities could be often increased by up to 100% more than the published capacity, as already evidenced by Fig. (15), which shows capacities approximately 40% above the present-day catalog. With automatic setting control applied to a crusher, reduction ratios could also be increased substantially on a given feed size distribution. By employing the present standard catalog format of capacities related to close side setting, no rational tool can be developed to accurately size the equipment.

Probably the most commonly used capacity and size graduation tables are those in the SME-AIME publication, "Mineral Processing Plant Design,"¹⁶ published in 1978. These are given for Rexnord's "Symons" crusher.

In early 1980, new capacities and installed power were given by the manufacturer for these machines. A 25-year survey of the literature, Table (3), shows changes to power for these units. Capacities for different sized machines also changed during this period.

TABLE (2) EXAMPLE OF CRUSHER APPLICATION ENGINEERING
INFORMATION USED BY ALLIS-CHALMERS PERSONNEL IN 1970

| COARSE CHAMBER – TWO ARM SPIDER | | | | | | | | | | | | | | | | | |
|---------------------------------|----------------------------------|-----------------|----------------------|--|------|-----|------|-----|-----|-----|-----|-----|-------|-------|-------|-----|-----|
| Nominal Crusher Size | Maximum Feed Size (Inches) | * Max. Hp | Ecc. Throw In. | CLOSE SIDE SETTING OF DISCHARGE OPENING (Inches) | | | | | | | | | | | | | |
| | | | | 1/4 | 5/16 | 3/8 | 7/16 | 1/2 | 5/8 | 3/4 | 7/8 | 1 | 1-1/4 | 1-1/2 | 1-3/4 | 2 | |
| 322 | 2x2x3 | 20 | 1/4 | 14 | 14.5 | 15 | 16 | 17 | | | | | | | | | |
| | | 25 | 3/8 | | | 22 | 25 | 27 | | | | | | | | | |
| | | 30 | 1/2 | | | | | 36 | | | | | | | | | |
| 736 | 4x4x6 | 75 | 5/8 | | | | | | | 88 | 92 | 96 | 100 | 107 | | | |
| | | 100 | 3/4 | | | | | | | | 111 | 115 | 120 | 130 | | | |
| | | 100 | 7/8 | | | | | | | | | | 139 | 143 | 152 | | |
| | | 125 | 1 | | | | | | | | | | | 160 | 175 | | |
| 945 | 6x6x8 | 100 | 5/8 | | | | | | | 98 | 101 | 105 | 109 | 117 | 125 | 133 | |
| | | 125 | 3/4 | | | | | | | | 126 | 130 | 135 | 143 | 152 | 160 | |
| | | 125 | 7/8 | | | | | | | | | 155 | 160 | 169 | 178 | 187 | |
| | | 150 | 1 | | | | | | | | | | 185 | 195 | 205 | 215 | |
| | | 200 | 1-1/4 | | | | | | | | | | | 247 | 259 | 270 | |
| 1051 | 6x6x9 | 125 | 3/4 | | | | | | | | 160 | 165 | 170 | 185 | 195 | 200 | |
| | | 150 | 7/8 | | | | | | | | | 190 | 195 | 205 | 210 | 220 | |
| | | 175 | 1 | | | | | | | | | | 235 | 245 | 250 | 260 | |
| | | 200 | 1-1/4 | | | | | | | | | | | 305 | 315 | 325 | |
| 1260 | 7x7x10 | 150 | 3/4 | | | | | | | | 200 | 210 | 220 | 230 | 240 | 250 | 260 |
| | | 200 | 1 | | | | | | | | | | 265 | 275 | 285 | 295 | 305 |
| | | 250 | 1-1/4 | | | | | | | | | | | 350 | 365 | 380 | 395 |
| | | 275 | 1-3/8 | | | | | | | | | | | | 415 | 430 | 445 |
| 1784 | 11x11x15 | 300 | 1 | | | | | | | | | | 440 | 450 | 460 | 470 | 480 |
| | | 350 | 1-1/4 | | | | | | | | | | | 555 | 565 | 575 | 585 |
| | | 400 | 1-1/2 | Recommended Max. and Min. Setting for each throw. | | | | | | | | | | 690 | 700 | 710 | |
| | | 450 | 1-3/4 | | | | | | | | | | | | | 870 | 885 |

*Maximum recommended HP.

Capacities shown are for open circuit operation and are expressed in short tons per hour and based on clean, dry, scalped, friable feed similar to limestone, and weighing 100 lbs. per cu. ft. with a work index of 13.0 or less.

TABLE (3)

| CATALOG HORSEPOWER — REXNORD "SYMONS" CRUSHERS | | | | |
|--|---------|--------------|------|--------------|
| Machine | 1955 | 1970 | 1973 | 1980 |
| 2 foot | 30 | 30-50 | 30 | 30 |
| 3 foot | 60 | 75-100 | 60 | 100 |
| 4 foot | 100 | 100-150 | 100 | 150 |
| 4-1/4 foot | 125-150 | 150-200 | 150 | 200 |
| 5 foot | N.A. | 200-250 | 200 | Discontinued |
| 5-1/2 foot HD | 200 | 200-250 | 200 | 250 |
| 5-1/2 foot XHD | 200 | Discontinued | — | — |
| 7 foot HD | 250-300 | 300-350 | 300 | 350 |
| 7 foot XHD | 300 | 300-350 | 350 | 400 |

For most manufacturers, nearly all the capacities quoted give the average size distribution for the discharged product as approximately 60% passing a size equivalent to the close side discharge setting. This is regardless of the eccentric throw or physical size of the machine. Because of such complications, designers and operators are referred to the factory for more accurate capacity and size distribution information. With some operators having reported product coarseness as low as 14% passing the close side setting, and with some only making a net average of 40% passing twice the close side setting on vast tonnages of easy-to-crush materials, the inexact nature of catalog information calls for a more accurate approach to crusher application.

Many engineers and some design organizations use standard published catalog capacities and size distributions in computer plant-design programs. Since most crusher manufacturers prefer people to seek factory advice for accurate application, information in such standard computer design programs can and has resulted in significant errors to plant operating performance.

One such plant, which cost several hundred million dollars to construct, compared actual performance data against design after more than one year's operation. This information is summarized in Fig. (19). In order to shield the plant's identity, capacity has been scaled to 100,000 tonnes per day. The numbers and sizes of crushers and screens are not given but have been scaled to the 100,000-tonnes-per-day figure. The plant was originally designed assuming it could make the required mill feed through 12.7 mm (1/2 inch) square screens operating 75 percent of total time or 18 hours per day in a 365-day year.

- (1) The plant was conservatively designed assuming a coarser feed material — 10% minus 12.7 mm versus the 47% actual. This was not, however, enough for the plant to make the design tonnage even when operating for a longer availability period. If the fines had been in the design propor-

tion, plant output would have only been approximately:

$$\frac{24 \times 2778}{0.9} \times 0.88 = 65,200$$

tonnes per day at 88 percent availability or 34,800 tonnes short of design.

- (2) The crushers are operating at lower capacities and at smaller settings than design.
- (3) Closed-circuit screen productivity is approximately half design (31 versus 56 TPH per square meter deck). This is because of a buildup in near size particles just above the screen aperture dimension.
- (4) The availability of the operating plant increases progressively from 80 percent for the secondary crushers to 88 percent for the tertiary screens. These screens are obviously the plant bottleneck, despite the fact that they are operating at a coarser aperture (14.3 mm) than design (12.7 mm).
- (5) The closed-circuit crushers are making a considerably coarser product (45 percent minus 12.7 mm compared to design, 66.5 percent) despite the fact that they are operating at 6.5 mm setting. The reader should remember that most catalog data would give a standard product for this setting as 60 percent passing 6.5 mm and 95 percent passing 12.7 mm. This coarse product is the reason for the bottleneck in this plant and could have been predicted if the facility had been designed using crusher power rates to calculate reduction ratios.

The crushers in this plant were automated using variable feed rate to control power drawn at a fixed close side setting. First-stage reduction crushers were running at 60 percent average connected power and the closed-circuit crushers were run at 95 percent average connected power. The equipment therefore was running near maximum capability. Obviously we need better design methods for crushing and screening plants if such errors are not to be repeated.

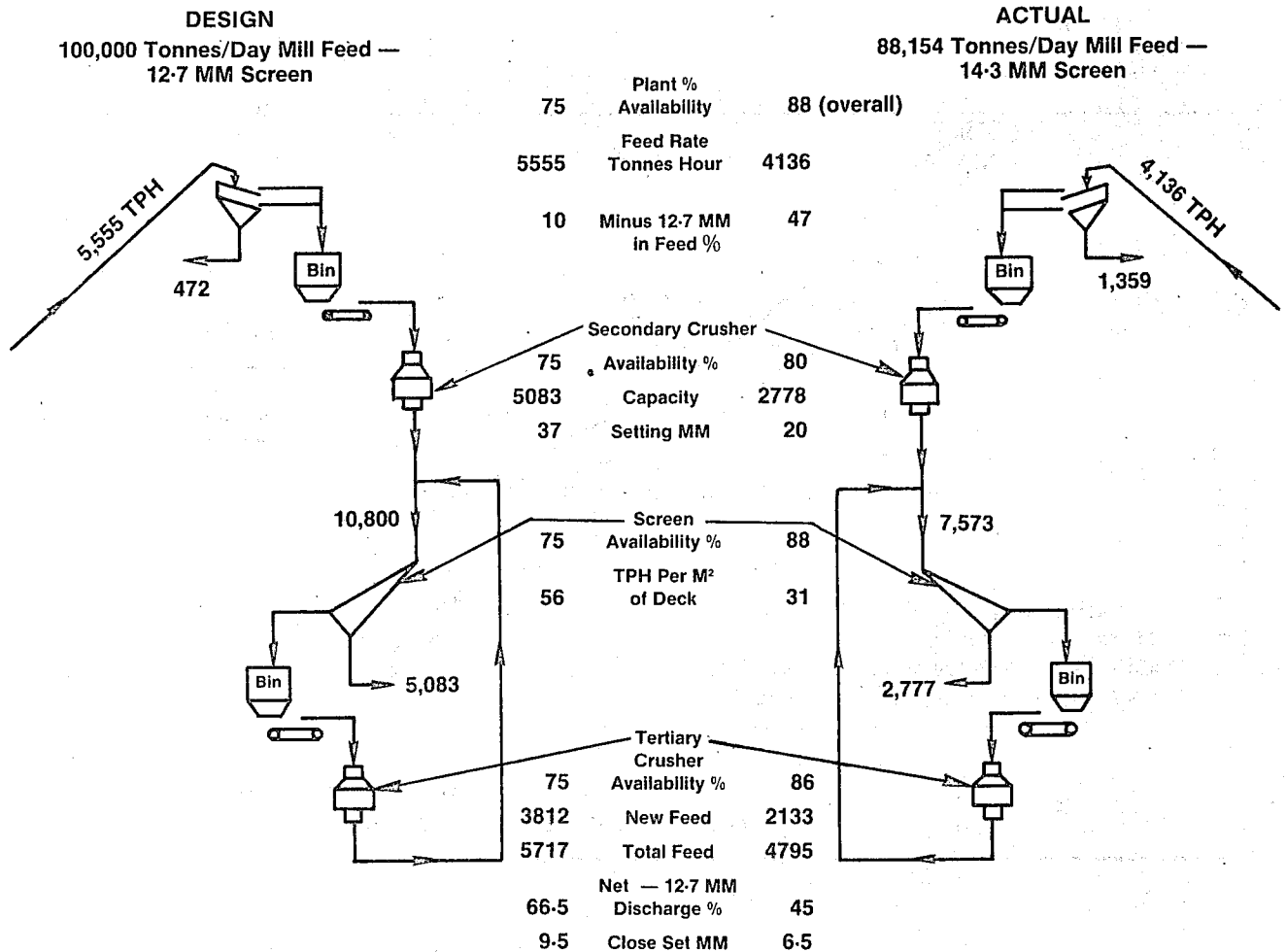


FIG. (19) COMPARISON OF DESIGN AND ACTUAL PLANT PERFORMANCE AFTER 12 MONTHS OPERATION

NEW DESIGN METHODS

By using automatic setting and feed rate controls on a crusher to maximize power drawn and optimize the power rate at each stage, significant improvements can be incorporated into any crushing circuit design.

In a recent paper¹² we described a method of designing a crushing plant using power drawn and power rate to define reduction ratios in each stage of crushing. The plant power and power rates were computed from a Bond calculation as applied to the crushing plant feed and output sizes. A comparison of the low and high energy configurations is shown in Fig. (20). Obviously the automated plant in Case B will cost much less to construct than Case A, so automation is an economic necessity.

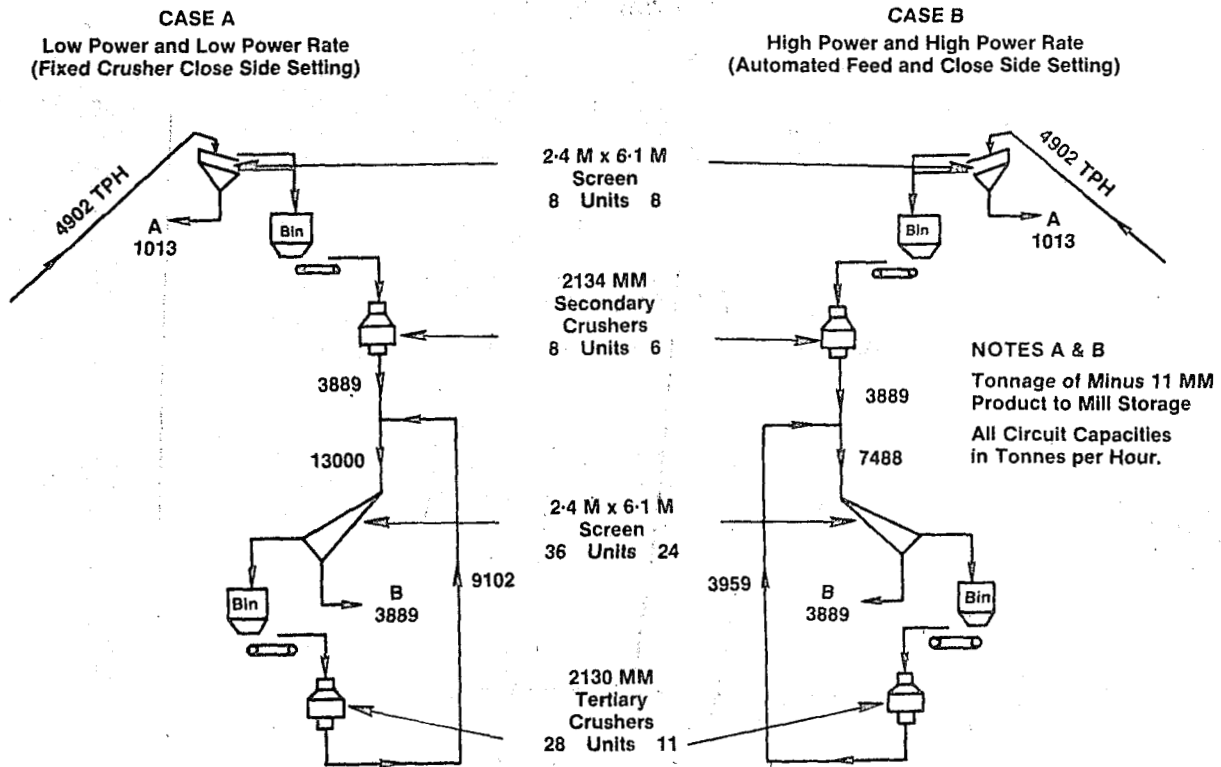
We would design this plant differently today using energy parameters from the pendulum impact tests for calculations. It would only be necessary to use the Bond feed and product size calculation if no pendulum results were available.

CRUSHING FINER TO REDUCE MILLING COSTS

This new high energy or power rate crushing brings a different perspective to comminution flow sheet selection.

Generally, up until the early 1960's the classical flow sheet for a beneficiation plant was primary crushing followed by two stages of cone crushing in closed or open circuit, making feed for rod mills, followed by ball mills. The rod mill was needed to reduce feed size to the ball mill because crushing plant output was normally coarser than 80% passing 10,000 microns. Such feed causes power inefficiency if fed directly to a ball mill. Even though the rod mill could be a relatively inefficient device for both energy and metal consumption, as was evidenced by Bond,¹⁷ it still made the overall circuit energy consumption more efficient.

Under the right operating conditions, high power rate crushing can bring mill feed size down to near 80% passing 7,000 microns and finer, which can be handled



more efficiently by ball mills. Based on average field observations, the crushers can do this for less than half the energy and between one-tenth and one-twentieth of the metal consumed in a rod mill.

It is, therefore, feasible to look at designing more efficient single stage ball mill circuits following two stages of fine crushing. The result will be an overall reduction in total applied crushing and milling energy for the same size reduction.

To make the most efficient use of both the crushing and grinding comminution energy, both reductions should be treated as dynamic components of the same system. When the feed to the grinding mills gets coarser and/or harder and the production rate drops, the crushing plant feed rate should be readjusted to a lower level to maximize power rate, which will flow on as a benefit helping to increase the mill output. Allis-Chalmers has patents granted and applied for on some such arrangements.

The advantages of such schemes will become more obvious when an ore of varying hardness is fed to the crushing and milling systems.

We will consider an ore with a ball mill work index varying between 16 and 13, feeding into a single stage ball mill operation with one million kilowatts per day consumed power. For the particular mill configuration,

a performance graph, Fig. (21), has been constructed according to Bond's methods.

Providing the crushing plant design allows for the machines to be fed continuously and the power on each crushing unit is maximized by adjusting both the feed rate and settings. The power drawn and reduction achieved to the grinding mill feed will be maximized.

The grinding mill output will vary considerably with the Work Index. If the feed size was 13,000 micrometers for the same grind production size, theoretical output from Fig. (21) would change from about 90,000 tons per day on the 13 Work Index down to 65,000 tons per day on the 16 Work Index.

Because of the superior energy efficiency of crushing over milling type processing, when the ore becomes harder in this system significant gains will be made if the feed rate to the crushing plant is reduced to closely match the mill production rate. If we consider the crushing plant runs at an average of 100,000 kilowatt hours per 20-hour day, the available energy for reduction will be:

$$\text{Soft ore Work Index 13} = \frac{100,000}{90,000} = 1.1 \text{ kwh/t}$$

$$\text{Hard ore Work Index 16} = \frac{100,000}{65,000} = 1.54 \text{ kwh/t}$$

For the purposes of this example, we will hypothesize that the the crushing index of the hard ore with the increased energy input of 1.54 kw/t reduces the ball mill

Bond Ball Mill Work Index (Short Tons)

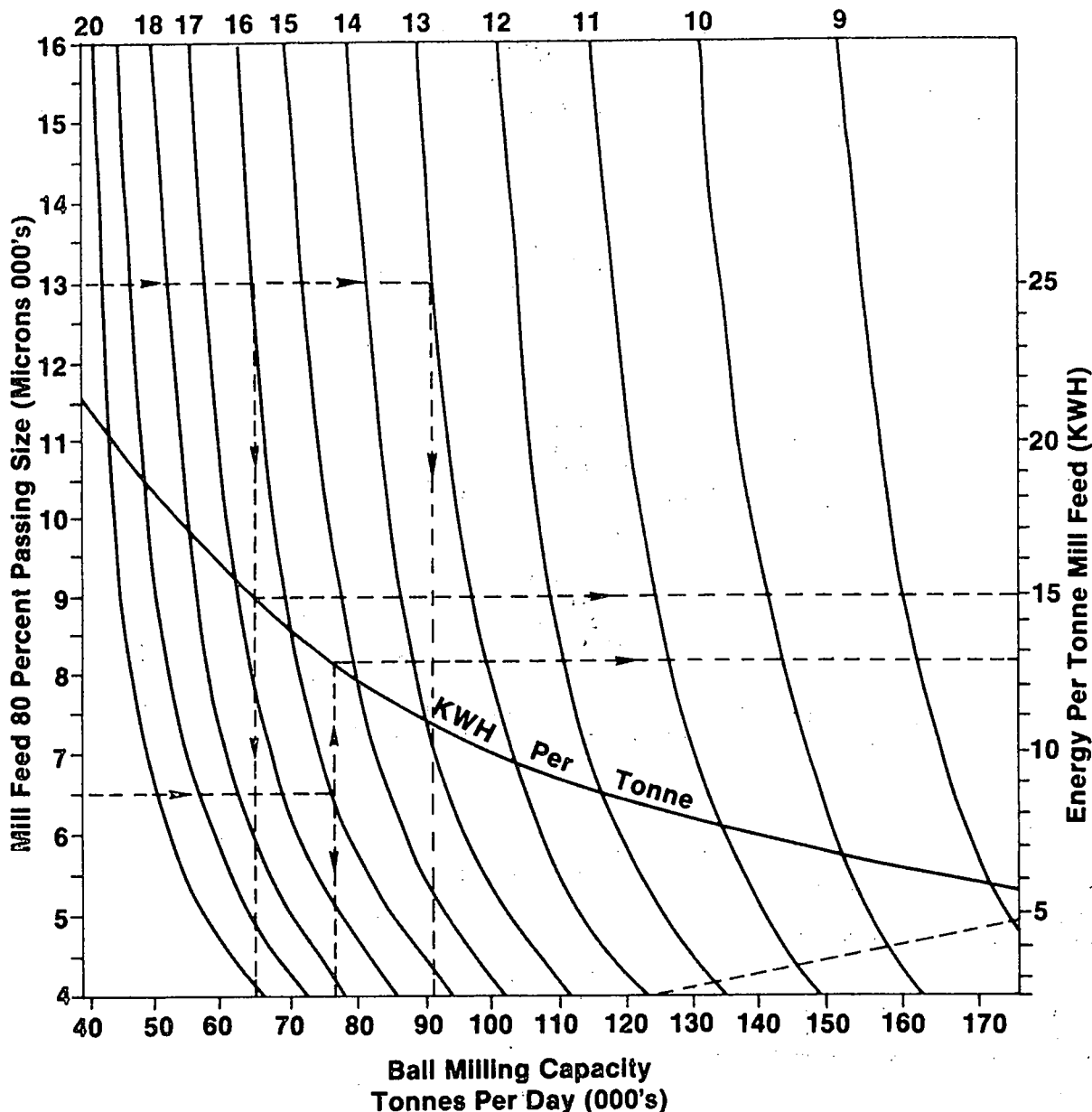


FIG. (21) CHANGING GRINDING MILL PRODUCTIVITY ACCORDING TO CHANGING WORK INDEX AND MILL FEED SIZE USING BOND FORMULAS FOR 1 MILLION KWH PER DAY CONSUMED POWER AT 95 PERCENT MOTOR EFFICIENCY

feed size to 6,500 micrometers. As a result, the mill output will increase with this reduced size to approximately 77,000 tons per day. The gain in production compared to the 13,000 micrometer feed will be:

$$\frac{(78,000 - 65,000)}{65,000} \times 100 = 20\%$$

The theoretical gain will actually be greater because the graph in Fig. (21) is constructed according to the Gates-Gaudin-Schuhmann size distribution used by Bond. We have already shown that this does not apply to crushing processes, which generate increased proportions of fines with higher energy input levels. As a con-

sequence of this, the actual gain is likely to be closer to 25% and the mill production increased to 65,000 x 1.25 = 81,250 tons per day.

Obviously, this will increase the capacity of the crushing plant and coarsen its reduction, again influencing mill output. Ideally a control system for the whole plant would balance both crushing and milling operations to maximize the benefits described.

Again, we might hypothesize that the crushing and milling output would fluctuate between rates of 78,000 and 90,000 tons per day instead of 65,000 and 90,000 tons per day. The advantages are obvious to all.

SUMMARY AND CONCLUSIONS

Whilst absolute conclusions cannot be drawn in every instance, the following trends and influences are clearly apparent from the discussion. Further studies in many areas are justified and are still the subject of continuing research programs.

- (1) As impact crushing energy is increased, an increasing proportion of finer sizes is produced. In general, the quantity of fines produced is proportional to the applied power rate (KWH/tonne).
- (2) A twin pendulum impact crushing device can generate product size distributions similar to those produced in commercial cone crushing operations.
- (3) By relating the net energy of pendulum crushing to the total energy in the commercial crusher, a comminution energy efficiency can be obtained for the commercial crusher.
- (4) The breakage size distribution obtained in the pendulum, crushing between two flat vertical plattens, is similar to that produced in a cone crusher. The effects of the circular chamber and the claims by some manufacturers that there is a special and most efficient angle to the horizontal for breakage in a cone crushing chamber are not corroborated by this study of fundamentals.
- (5) It is a claim for some designs of cone crushers that the eccentric throw has some mystical influence on the capacity and size distribution produced by a particular machine. The pendulum tests supported by field observations prove conclusively that reduction and size distribution of products are simply related to the energy applied to the material. For a fixed application of power, increasing eccentric throw will reduce the force available to crush. This lowers the available power rate and, hence, the reduction ratio. The size distribution appears to be independent of the number of impacts causing this energy application.
- (6) The pendulum results prove that the size distribution produced from crushing processes is a natural breakage pattern characteristic of the material and to a lesser extent the mode of breakage.
- (7) The size distribution occurring in crushing processes cannot be described by the Gates-Gaudin-Schuhmann log size versus log cumulative percent passing used by Bond or the Rosin-Rammler relationship. There appears to be little correlation in the finer size ranges of products actually occurring and those predicted by such relationships. New mathematical expressions are needed to define crusher energy size relationships.
- (8) This study of fundamentals does not support the statement of some crusher manufacturers that crushers operate inefficiently at higher energy input levels. The production of coarser sizes will be less efficient, should this be the desired product for a process such as crushed stone production, if power rates greater than that required to maximize that coarse size are applied.
- (9) Crusher power rate, which is influenced by the geometry and eccentric throw of the crushing chamber as well as the feed size and hardness of the processed material, will determine reduction and size distribution in the crusher product. This product is related to the close side setting only if this is considered to control crushing energy application.
- (10) Automatic setting regulation as applied to cone crushers can increase the average power drawn and maximize power rate along with reduction ratio. The phenomena can be studied with the pendulum impact test apparatus.
- (11) Crusher productivity in quantities of a fixed size distribution is a function of power drawn as opposed to physical machine size. From the results of the pendulum tests, it is possible to visualize a small machine drawing more power outproducing a larger machine drawing less power.
- (12) Energy utilization by crushing and milling machinery is different. As a general conclusion, this is particularly so with rod mills and, to a lesser extent, ball mills. High power rate crushing resulting in increased fines, can bring significant energy conservation to comminution circuits using both forms of breakage. The pendulum test device will allow us to quantify potential gains in the laboratory.
- (13) Crushing machines consume less wear metal than grinding mills. This is especially so in the case of rod mills which compete in the fine crushing area. Because wear metal consumption is related to energy consumed to reduce material to a given size, any energy reduction using high power rate crushing will reduce such metal consumption. The pendulum impact test will give a measure of potential gain from this area in the laboratory.
- (14) Because the pendulum test can be used to predict the size distribution from cone crushers, finite calculations can now be made for the following parameters, some of which have often been decided by "Rules of Thumb":
 - (a) The number of crusher reduction stages.
 - (b) Number of crushing machines and their connected power.
 - (c) Circuit loadings.
 - (d) Screen requirements.
 - (e) Size distribution of plant output.

This methodology will give more accurate results than designs using standardized capacity tables and product size distribution curves.
- (15) The pendulum tester can be used to run tests on drill core sized material so that plant designs can be more accurately formulated from geological

exploration data. Pilot plant tests, which are often impractical to run because of the lack of sufficient sample material, are not now a necessity for feasibility and scale-up studies.

- (16) The guestimation type comparisons of flow sheets using conventional crushing and grinding processes versus autogenous milling can now be more accurately made. The high power rate crushing process that has been studied here is not normally part of such engineering evaluations. It can now be made so, quickly and relatively inexpensively, by using the pendulum impact device.
- (17) As a result of these studies, it is postulated that the current method of running crushing plants at maximum capacity through a given screen size will not minimize energy and wear metal costs in beneficiation plants employing downstream grinding mills. Crushing and milling circuit control systems that balance the crushing plant output to the grinding mill capacity will maximize plant capacity at the lowest production cost.
- (18) Properly applied, the technology described will reduce the magnitude of the over-design parameters used in most comminution plant design calculations. This should reduce the capital costs of these plants.

We hope this discussion has contributed to a better understanding of the crushing process and how it fits into the spectrum of comminution processing machines. Hopefully, there will be contributions from other manufacturers and research organizations to expand on this subject.

ACKNOWLEDGEMENTS

The program that initiated research into alternative testing methods was formulated by Allis-Chalmers' Comminution Task Force Committee, which must take credit for the development. Many people were involved over the years in the development and the following deserve special mention: D. C. Moore, who selected the equipment and formulated methods and procedures for its use; E. S. Burkhardt, F. P. Faulkner, C. J. Terry and K. S. Joung, all of whom were or are associated with Allis-Chalmers Corporation. We would also like to express gratitude to the many commercial operations and their employees who gave, generously, time and information to make this research program a success.

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