

PART I. CRYSTALLOGRAPHY

GENERAL MORPHOLOGICAL RELATIONS OF CRYSTALS

5. Crystallography.— The subject of Crystallography includes the description of the characters of crystals in general; of the various forms of crystals and their division into classes and systems; of the methods of studying crystals, including the determination of the mathematical relations of their faces, and the measurement of the angles between them; finally, a description of compound or twin crystals, of irregularities in crystals, of crystalline aggregates, and of pseudomorphous crystals.

6. Definition of a Crystal.— *A crystal* is the regular polyhedral form, bounded by smooth surfaces, which is assumed by a chemical compound, under the action of its intermolecular forces, when passing, under suitable conditions, from the state of a liquid or gas to that of a solid.*

As expressed in the foregoing definition, a crystal is characterized, first, by its definite internal molecular structure, and, second, by its external form. A crystal is the *normal* form of a mineral species, as of all solid chemical compounds; but the conditions suitable for the formation of a crystal of ideal perfection in symmetry of form and smoothness of surface are never fully realized. Further, many species usually occur not in distinct crystals, but in massive form, and in some exceptional cases the definite molecular structure is absent.

7. Molecular Structure in General.— By definite molecular structure is meant the special arrangement which the physical units, called *molecules*,† assume under the action of the forces exerted between them during the formation of the solid. Some remarks are given in a later article (p. 22 *et seq.*) in regard to the kinds of molecular arrangement theoretically possible, and their relation to the symmetry of the different systems and classes of crystals.

The definite molecular structure is the essential character of a crystal, and the external form is only one of the ways, although the most important, in which this structure is manifested. Thus it is found that all similar directions in a crystal, or a fragment of a crystal, have like physical characters,‡

* In its original signification the term *crystal* was applied only to crystals of quartz, which the ancient philosophers believed to be *water* congealed by intense cold. Hence the term, from *κρύσταλλος*, *ice*.

† Recent studies, particularly those made by the use of the X-ray, would indicate that the unit of crystalline structure is the atom rather than the molecule. The grouping of the atoms to form a molecule is extended in the analogous grouping of the molecules to form a crystal.

‡ This subject is further elucidated in the chapter devoted to Physical Mineralogy, where it is also shown that, with respect to many, but not all, of the physical characters, the converse of this proposition is true, *viz.*, that unlike directions in a crystal have in general unlike properties.

as of elasticity, cohesion, action on light, etc. This is clearly shown by the cleavage, or natural tendency to fracture in certain directions, yielding more or less smooth surfaces; as the cubic cleavage of galena, or the rhombohedral cleavage of calcite. It is evident, therefore, that a small crystal differs from a large one only in size, and that a fragment of a crystal is itself essentially a crystal in all its physical relations, though showing no crystalline faces.

Further, the external form without the corresponding molecular structure does not make a crystal of a solid. A model of glass or wood is obviously not a crystal, though having its external form, because there is no relation between form and structure. Also, an octahedron of malachite, having the form of the crystal of cuprite from which it has been derived by chemical alteration, is not a crystal of malachite, but what is known as a pseudomorph (see Art. 478) of malachite after cuprite.

On the other hand, if the natural external faces are wanting, the solid is not called a crystal. A cleavage octahedron of fluorite and a cleavage rhombohedron of calcite are not properly crystals, because the surfaces have been yielded by fracture and not by the natural molecular growth of the crystal.

8. Crystalline and Amorphous. — When a mineral shows no external crystalline form, it is said to be *massive*. It may, however, have a definite molecular structure, and then it is said to be *crystalline*. If this structure, as shown by the cleavage, or by optical means, is the same in all parallel directions through the mass, it is described as a single individual. If it varies from grain to grain, or fiber to fiber, it is said to be a *crystalline aggregate*,* since it is in fact made up of a multitude of individuals.

Thus in a granular mass of galena or calcite, it may be possible to separate the fragments from one another, each with its characteristic cubic, or rhombohedral, cleavage. Even if the individuals are so small that they cannot be separated, yet the cleavage, and hence the crystalline structure, may be evident from the spangling of a freshly broken surface, as with fine-grained statuary marble. Or, again, this aggregate structure may be so fine that the crystalline structure can only be resolved by optical methods with the aid of the microscope. In all these cases, the structure is said to be *crystalline*.

If optical means show a more or less distinct crystalline structure, which, however, cannot be resolved into individuals, the mass is said to be *cryptocrystalline*; this is true of some massive varieties of quartz.

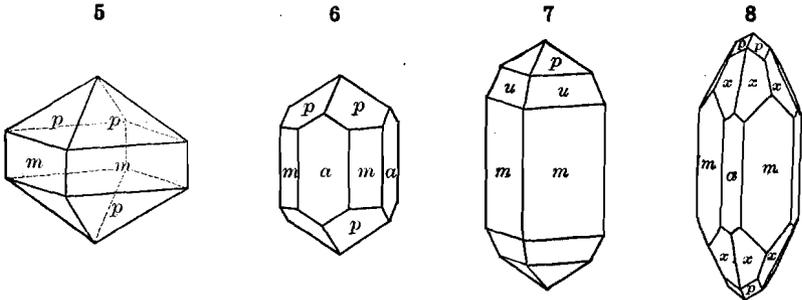
If the definite molecular structure is entirely wanting, and all directions in the mass are sensibly the same, the substance is said to be *amorphous*. This is true of a piece of glass, and nearly so of opal. The amorphous state is rare among minerals.

A piece of feldspar which has been fused and cooled suddenly may be in the glass-like amorphous condition as regards absence of definite molecular structure. But even in such cases there is a tendency to go over into the crystalline condition by molecular rearrangement. A transparent amorphous mass of arsenic trioxide (As_2O_3), formed by fusion, becomes opaque and crystalline after a time. Similarly the steel beams of a railroad bridge may gradually become crystalline and thus lose some of their original strength because of the molecular rearrangement made possible by the vibrations caused by the frequent jar of passing trains. The microscopic study of rocks reveals many cases in which an analogous change in molecular structure has taken place in a solid mass, as caused, for example, by great pressure.

* The consideration of the various forms of crystalline aggregates is postponed to the end of the present chapter.

There is no truth in this.

12. Diversity of Form, or Habit. — While in the crystals of a given species there is constancy of angle between like faces, the forms of the crystals may be exceedingly diverse. The accompanying figures (5–8) are examples of a few of the forms of the species zircon. There is hardly any limit to the number of faces which may occur, and as their relative size changes, the *habit*, as it is called, may vary indefinitely.



Zircon

13. Diversity of Size. — Crystals occur of all sizes, from the merest microscopic point to a yard or more in diameter. It is important to understand, however, that in a minute crystal the development is as complete as with a large one. Indeed the highest perfection of form and transparency is found only in crystals of small size.

A single crystal of quartz, now at Milan, is three and a quarter feet long and five and a half in circumference, and its weight is estimated at eight hundred and seventy pounds. A single cavity in a vein of quartz near the Tiefen Glacier, in Switzerland, discovered in 1867, afforded smoky quartz crystals, a considerable number of which had a weight of 200 to 250 pounds. A gigantic beryl from Acworth, New Hampshire, measured four feet in length and two and a half in circumference; another, from Grafton, was over four feet long, and thirty-two inches in one of its diameters, and weighed about two and a half tons.

14. Symmetry in General. — The faces of a crystal are arranged according to certain laws of symmetry, and this symmetry is the natural basis of the division of crystals into systems and classes. The symmetry may be defined in relation to (1) a *plane of symmetry*, (2) an *axis of symmetry*, and (3) a *center of symmetry*.

These different kinds of symmetry may, or may not, be combined in the same crystal. It will be shown later that there is one class, the crystals of which have neither center, axis, nor plane of symmetry; another where there is only a center of symmetry. On the other hand, some classes have all these elements of symmetry represented.

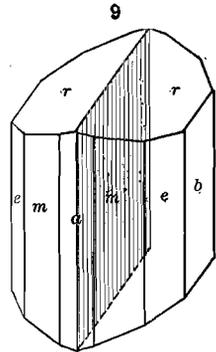
15. Planes of Symmetry. — A solid is said to be geometrically * symmetrical with reference to a plane of symmetry when for each face, edge, or solid angle there is another similar face, edge, or angle which has a like position with reference to this plane. Thus it is obvious that the crystal of amphibole, shown in Fig. 9, is symmetrical with reference to the central plane of symmetry indicated by the shading.

* The relation between the ideal geometrical symmetry and the actual crystallographic symmetry is discussed in Art. 18.

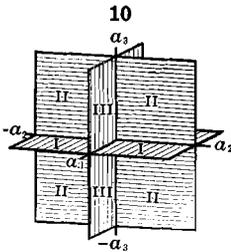
In the ideal crystal this symmetry is *right symmetry* in the geometrical sense, where every point on the one side of the plane of symmetry has a corresponding point at equal distances on the other side, measured on a line normal to it. In other words, in the ideal geometrical symmetry, one half of the crystal is the exact *mirror-image* of the other half.

A crystal may have as many as nine planes of symmetry, three of one set and six of another, as is illustrated by the cube* (Fig. 16). Here the planes of the first set pass through the crystal parallel to the cubic faces; they are shown in Fig. 10. The planes of the second set join the opposite cubic edges; they are shown in Fig. 11.

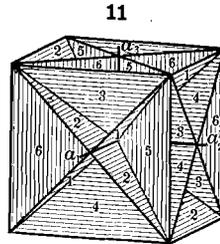
16. Axes of Symmetry. — If a solid can be revolved through a certain number of degrees about some line as an axis, with the result that it again occupies precisely the same position in space as at first, that axis is said to be an axis of symmetry. There are four different kinds of axes of symmetry among crystals; they are defined according to the number of times which the crystal repeats itself in appearance during a complete revolution of 360° .



Amphibole



Symmetry Planes in the Cube



(a) A crystal is said to have an axis of *binary*, or *twofold*, symmetry when a revolution of 180° produces the result named above; in other words, when it repeats itself twice in a complete revolution. This is true of the crystal shown in Fig. 12 with respect to the vertical axis (and indeed each of the horizontal axes also).

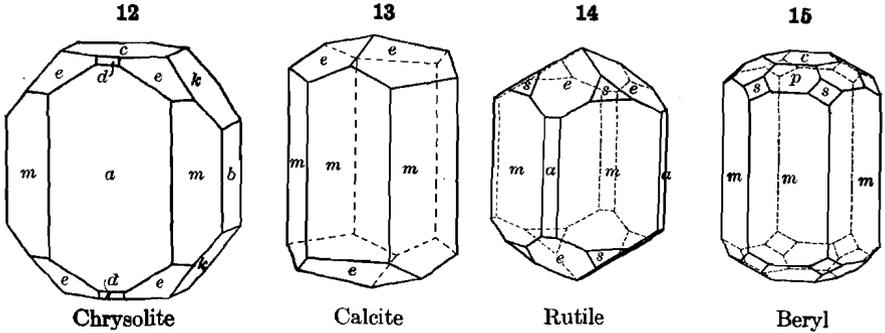
(b) A crystal has an axis of *trigonal*, or *threefold*, symmetry when a revolution of 120° is needed; that is, when it repeats itself three times in a complete revolution. The vertical axis of the crystal shown in Fig. 13 is an axis of trigonal symmetry.

(c) A crystal has an axis of *tetragonal*, or *fourfold*, symmetry when a revolution of 90° is called for; in other words, when it repeats itself four times in a complete revolution. The vertical axis in the crystal shown in Fig. 14 is such an axis.

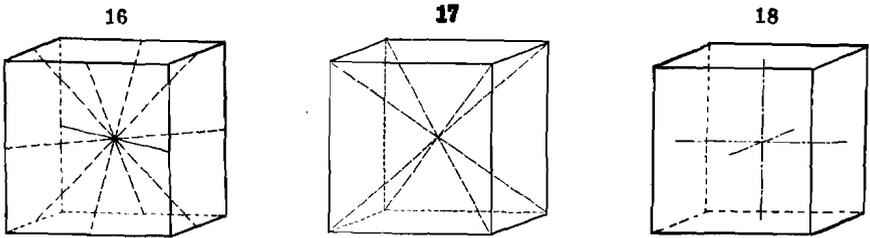
(d) Finally, a crystal has an axis of *hexagonal*, or *sixfold*, symmetry when a revolution of 60° is called for; in other words, when it repeats itself six times in a complete revolution. This is illustrated by Fig. 15.

* This is the cube of the normal class of the isometric system.

The different kinds of symmetry axes are sometimes known as *diad*, *triad*, *tetrad* and *hexad* axes.

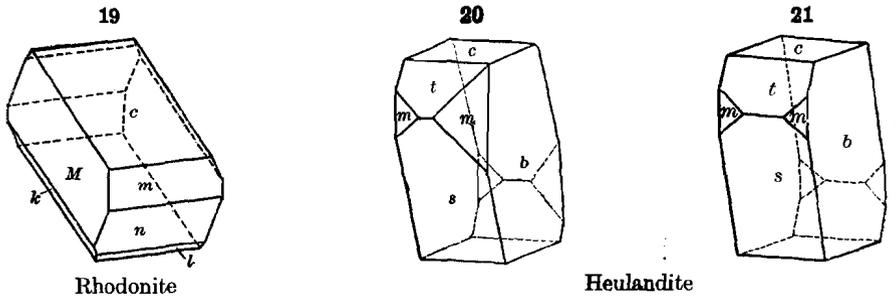


The cube * illustrates three of the four possible kinds of symmetry with respect to axes of symmetry. It has six axes of *binary* symmetry joining the middle points of opposite edges (Fig. 16). It has four axes of *trigonal* symmetry, joining the opposite solid angles (Fig. 17). It has, finally, three axes of *tetragonal* symmetry joining the middle points of opposite faces (Fig. 18).



Symmetry Axes in the Cube

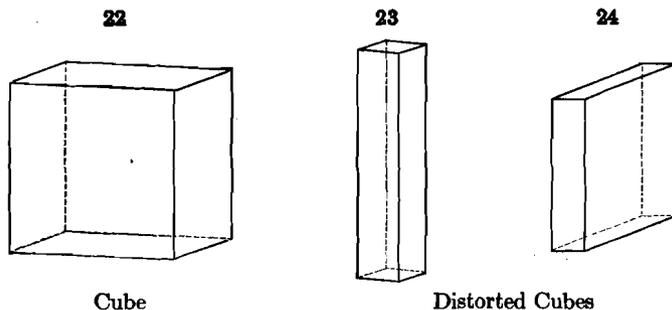
17. Center of Symmetry. — Most crystals, besides planes and axes of symmetry, have also a center of symmetry. On the other hand, a crystal, though possessing neither plane nor axis of symmetry, may yet be sym-



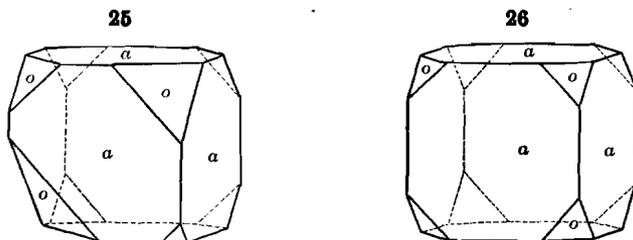
metrical with reference to a point, its center. This last is true of the triclinic crystal shown in Fig. 19, in which it follows that every face, edge, and solid angle has a face, edge, and angle similar to it in the opposite half of the crystal.

* This is again the cube of the normal class of the isometric system.

18. Relation of Geometrical to Crystallographic Symmetry. — Since the symmetry in the arrangement of the faces of a crystal is an expression of the internal molecular structure, which in general is alike in all parallel directions, the *relative size* of the faces and their *distance* from the plane or axis of symmetry are of no moment, their *angular position* alone is essential. The crystal represented in Fig. 20, although its faces show an unequal development, has in the crystallographic sense as truly a vertical plane of symmetry (parallel to the face *b*) as the ideally developed crystal shown in Fig. 21. The strict geometrical definition of symmetry would, however, apply only to the second crystal.*



Also in a normal cube (Fig. 22) the three central planes parallel to each pair of cubic faces are like planes of symmetry, as stated in Art. 15. But a crystal is still crystallographically a cube, though deviating widely from the requirements of the strict geometrical definition, as shown in Figs. 23, 24, if only it can be proved, *e.g.*, by cleavage, by the physical nature of the faces, or by optical means, that the three pairs of faces are *like* faces, independently of their size, or, in other words, that the molecular structure is the same in the three directions normal to them.



Cube and Octahedron

Further, in the case of a normal cube, a face of an octahedron on any solid angle requires, as explained beyond, similar faces on the other angles. It is *not* necessary, however, that these eight faces should be of equal size, for in the crystallographic sense Fig. 25 is as truly symmetrical with reference to the planes named as Fig. 26.

* It is to be noted that the perspective figures of crystals usually show the geometrically ideal form, in which like faces, edges, and angles have the same shape, size, and position. In other words, the ideal crystal is uniformly represented as having the symmetry called for by the strict geometrical definition.

19. On the other hand, the molecular and hence the crystallographic symmetry is not always that which the geometrical form would suggest. Thus, deferring for the moment the consideration of pseudo-symmetry, an illustration of the fact stated is afforded by the cube. It has already been implied and will be fully explained later that while the cube of the normal class of the isometric system has the symmetry described in Arts. 15, 16, a cube of the same geometrical form but belonging molecularly, for example, to the tetrahedral class, has no planes of symmetry parallel to the faces but only the six diagonal planes; further, though the four axes shown in Fig. 17 are still axes of trigonal symmetry, the cubic axes (Fig. 18) are axes of binary symmetry only, and there are no axes of symmetry corresponding to those represented in Fig. 16. Other more complex cases will be described later.

Further, a crystal having interfacial angles of 90° is not necessarily a cube: in other words, the angular relations of the faces do not show in this case whether the figure is bounded by six like faces; or whether only four are alike and the other pair unlike; or, finally, whether there are three pairs of unlike faces. The question must be decided, in such cases, by the molecular structure as indicated by the physical nature of the surfaces, by the cleavage, or by other physical characters, as pyro-electricity, those connected with light phenomena, etc.

Still, again, the student will learn later that the decision reached in regard to the symmetry to which a crystal belongs, based upon the *distribution* of the faces, is only preliminary and approximate, and before being finally accepted it must be confirmed, first, by accurate measurements, and, second, by a minute study of the other physical characters.

The method based upon the physical characters, which gives most conclusive results and admits of the widest application, is the skillful etching of the surface of the crystal by some appropriate solvent. By this means there are, in general, produced upon it minute depressions the shape of which conforms to the symmetry in the arrangement of the molecules. This process, which is in part essentially one involving the dissection of the molecular structure, is more particularly discussed in the chapter on Physical Mineralogy.

20. **Pseudo-symmetry.** — The crystals of certain species approximate closely in angle, and therefore in apparent symmetry, to the requirements of a system higher in symmetry than that to which they actually belong: they are then said to exhibit *pseudo-symmetry*. Numerous examples are given under the different systems. Thus the micas have been shown to be truly monoclinic in crystallization, though in angle they seem to be in some cases rhombohedral, in others orthorhombic.

It will be shown later that compound, or twin, crystals may also simulate by their regular grouping a higher grade of symmetry than that which belongs to the single crystal. Such crystals also exhibit pseudo-symmetry and are specifically called *mimetic*. Thus aragonite is an example of an orthorhombic species, whose crystals often imitate by twinning those of the hexagonal system.* Again, a highly complex twinned crystal of the monoclinic species, phillipsite, may have nearly the form of a rhombic dodecahedron of the isometric system. This kind of pseudo-symmetry also occurs among the classes of a single system, since a crystal belonging to a class of low symmetry may by twinning gain the geometrical symmetry of the corresponding

* The terms *pseudo-hexagonal*, etc., used in this and similar cases explain themselves.

form of the normal class. This is illustrated by a twinned crystal of scheelite like that figured (Fig. 416) in the chapter on twin crystals.

Pseudo-symmetry of still another kind, where there is an imitation of the symmetry of another system of lower grade, is particularly common in crystals of the isometric system (*e.g.*, gold, copper). The result is reached in such cases by an abnormal development of "distortion" in the direction of certain axes of symmetry. This subject is discussed and illustrated on a later page.

21. Possible Classes of Symmetry. — The theoretical consideration of the different kinds of symmetry possible among crystals built up of like molecules, as explained in Arts. 30–32, has led to the conclusion that there are thirty-two (32) types in all, differing with respect to the combination of the different symmetry elements just described. Of these thirty-two natural classes among crystals based upon their symmetry, seven classes include by far the larger number of crystallized minerals. Besides these, some thirteen or fourteen others are distinctly represented, though several of these are of rare occurrence. The remaining classes, with possibly one or two exceptions, are known among the crystallized salts made in the laboratory. The characters of each of the thirty-two classes are given under the discussion of the several crystalline systems.

22. Crystallographic Axes. — In the description of a crystal, especially as regards the position of its faces, it is found convenient to assume, after the methods of analytical geometry, certain lines passing through the center of the ideal crystal, as a basis of reference. (See further Art. 34 *et seq.*)

These lines are called the *crystallographic axes*. Their direction is to a greater or less extent fixed by the symmetry of the crystals, for an axis of symmetry is in almost all cases* a possible crystallographic axis. Further, the unit lengths assigned to these axes are fixed sometimes by the symmetry, sometimes by the position of the faces assumed as fundamental, *i.e.*, the unit forms in the sense defined later. The broken lines shown in Fig. 18 are the crystallographic axes to which the cubic faces are referred.

23. Systems of Crystallization. — The thirty-two possible crystal classes which are distinguished from one another by their symmetry, are classified in this work under six systems, each characterized by the relative lengths and inclinations of the assumed crystallographic axes. These are as follows:

I. ISOMETRIC SYSTEM. Three equal axes at right angles to each other.

II. TETRAGONAL SYSTEM. Three axes at right angles to each other, two of them — the horizontal axes — equal, the third — the vertical axis — longer or shorter.

III. HEXAGONAL SYSTEM. Four axes, three equal horizontal axes in one plane intersecting at angles of 60° , and a vertical axis at right angles to this plane and longer or shorter.

IV. ORTHORHOMBIC SYSTEM. Three axes at right angles to each other, but all of different lengths.

V. MONOCLINIC SYSTEM. Three axes unequal in length, and having one of their intersections oblique, the two other intersections equal to 90° .

VI. TRICLINIC SYSTEM. Three unequal axes with mutually oblique intersections.

* Exceptions are found in the isometric system, where the axes must necessarily be the axes of tetragonal symmetry (Fig. 18), and cannot be those of binary or trigonal symmetry (Figs. 16, 17).

24. Each one of the six systems, as will be understood from Art. 21, embraces several classes differing among themselves in their symmetry. One of these classes is conveniently called the *normal* class, since it is in general the common one, and since further it exhibits the highest degree of symmetry possible for the given system, while the others are lower in grade of symmetry.

It is important to note that the classes comprised within a given system are at once essentially connected together by their common optical characters, and in general separated * from those of the other systems in the same way.

Below is given a list of the six systems together with their subordinate classes, thirty-two in all. The order and the names given first are those that are used in this book while in the following parentheses are given other equivalent names that are also in common use. Under nearly all of the classes it is possible to give the name of a mineral or an artificial compound whose crystals serve to illustrate the characters of that particular class. There is some slight variation between different authors in the order in which the crystal systems and classes are considered but in the main essentials all modern discussions of crystallography are uniform.

ISOMETRIC SYSTEM

(*Regular, Cubic System*)

1. NORMAL CLASS. (Hexoctahedral. Holohedral.) Galena Type.
2. PYRITOHEDRAL CLASS. (Dyakisdodecahedral. Pentagonal Hemihedral.) Pyrite Type.
3. TETRAHEDRAL CLASS. (Hextetrahedral. Tetrahedral Hemihedral.) Tetrahedrite Type.
4. PLAGIOHEDRAL CLASS. (Pentagonal Icositetrahedral. Plagihedral Hemihedral.) Cuprite Type.
5. TETARTOHEDRAL CLASS. (Tetrahedral Pentagonal Dodecahedral.) Sodium Chlorate Type.

TETRAGONAL SYSTEM

6. NORMAL CLASS. (Ditetragonal Bipyramidal. Holohedral.) Zircon Type.
7. HEMIMORPHIC CLASS. (Ditetragonal Pyramidal. Holohedral Hemimorphic.) Iodosuccinimide Type.
8. TRIPYRAMIDAL CLASS. (Tetragonal Bipyramidal. Pyramidal Hemihedral.) Scheelite Type.
9. PYRAMIDAL-HEMIMORPHIC CLASS. (Tetragonal Pyramidal. Hemihedral Hemimorphic.) Wulfenite Type.
10. SPHENOIDAL CLASS. (Tetragonal Sphenoidal. Sphenoidal Hemihedral. Scalenohedral.) Chalcopyrite Type.
11. TRAPEZOHEDRAL CLASS. (Tetragonal Trapezohedral. Trapezohedral Hemihedral.) Nickel Sulphate Type.
12. TETARTOHEDRAL CLASS. (Tetragonal Bisphenoidal.) Artif. $2\text{CaO} \cdot \text{Al}_2\text{O}_3 \cdot \text{SiO}_2$ Type.

* Crystals of the tetragonal and hexagonal systems are alike in being optically uniaxial; but the crystals of all the other systems have distinguishing optical characters.

HEXAGONAL SYSTEM

A. HEXAGONAL DIVISION

13. NORMAL CLASS. (Dihexagonal Bipyramidal. Holohedral.) Beryl Type.
14. HEMIMORPHIC CLASS. (Dihexagonal Pyramidal. Holohedral Hemimorphic.) Zincite Type.
15. TRIPYRAMIDAL CLASS. (Hexagonal Bipyramidal. Pyramidal Hemihedral.) Apatite Type.
16. PYRAMIDAL-HEMIMORPHIC CLASS. (Hexagonal Pyramidal. Pyramidal Hemihedral Hemimorphic.) Nephelite Type.
17. TRAPEZOHEDRAL CLASS. (Hexagonal Trapezohedral. Trapezohedral Hemihedral.) β -Quartz Type.

B. TRIGONAL OR RHOMBOHEDRAL DIVISION

(Trigonal System)

18. TRIGONAL CLASS. (Ditrigonal Bipyramidal. Trigonal Hemihedral.) Benitoite Type.
19. RHOMBOHEDRAL CLASS. (Ditrigonal Scalenohedral. Rhombohedral Hemihedral.) Calcite Type.
20. RHOMBOHEDRAL HEMIMORPHIC CLASS. (Ditrigonal Pyramidal. Trigonal Hemihedral Hemimorphic.) Tourmaline Type.
21. TRI-RHOMBOHEDRAL CLASS. (Rhombohedral. Rhombohedral Tetartohedral.) Phenacite Type.
22. TRAPEZOHEDRAL CLASS. (Trigonal Trapezohedral. Trapezohedral Tetartohedral.) Quartz Type.
23. (Trigonal Bipyramidal. Trigonal Tetartohedral.)
24. (Trigonal Pyramidal. Trigonal Tetartohedral Hemimorphic.) Sodium Periodate Type.

ORTHORHOMBIC SYSTEM

(Rhombic or Prismatic System)

25. NORMAL CLASS. (Orthorhombic Bipyramidal. Holohedral.) Barite Type.
26. HEMIMORPHIC CLASS. (Orthorhombic Pyramidal.) Calamine Type.
27. SPHENOIDAL CLASS. (Orthorhombic Bisphenoidal.) Epsomite Type.

MONOCLINIC SYSTEM

(Oblique System)

28. NORMAL CLASS. (Prismatic. Holohedral.) Gypsum Type.
29. HEMIMORPHIC CLASS. (Sphenoidal.) Tartaric Acid Type.
30. CLINOEDRAL CLASS. (Domatic. Hemihedral.) Clinohedrite Type.

TRICLINIC SYSTEM

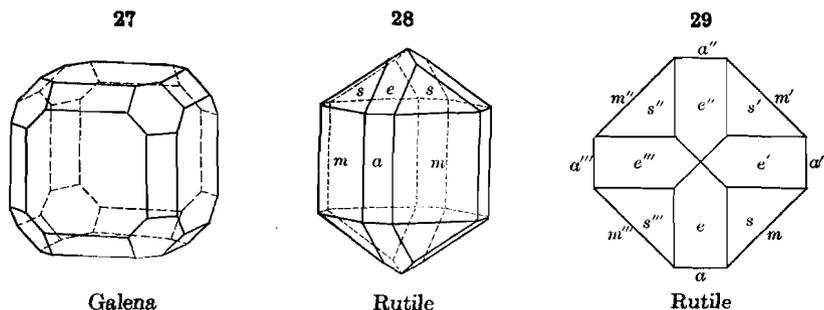
(Anorthic System)

31. NORMAL CLASS. (Holohedral. Pinacoidal.) Axinite Type.
32. ASYMMETRIC CLASS. (Hemihedral.) Clacium Thiosulphate Type.

25. Symmetry of the Systems. — In the paragraphs immediately following, a synopsis is given of the symmetry of the *normal class* of each of the different systems, and also that of one subordinate class of the hexagonal system, which is of so great importance that it is also often conveniently treated as a sub-system even when, as in this work, the forms are referred to the same axes as those of the strictly hexagonal type — a usage not adopted by all authors.

I. ISOMETRIC SYSTEM. Three like axial * planes of symmetry (principal planes) parallel to the cubic faces, and fixing by their intersection the crystallographic axes; six like diagonal planes of symmetry, passing through each opposite pair of cubic edges, and hence parallel to the faces of the rhombic dodecahedron.

Further, three like axes of tetragonal symmetry, the crystallographic axes normal to the faces of the cube; four like diagonal axes of trigonal symmetry, normal to the faces of the octahedron; and six like diagonal axes of binary symmetry, normal to the faces of the dodecahedron. There is also obviously a center of symmetry.† These relations are illustrated by Fig. 27 also by Fig. 35; further by Figs. 92 to 125.



II. TETRAGONAL SYSTEM. Three axial planes of symmetry: of these, two are like planes intersecting at 90° in a line which is the vertical crystallographic axis, and the third plane (a principal plane) is normal to them and hence contains the horizontal axes. There are also two diagonal planes of symmetry, intersecting in the vertical axis and meeting the two axial planes at angles of 45° .

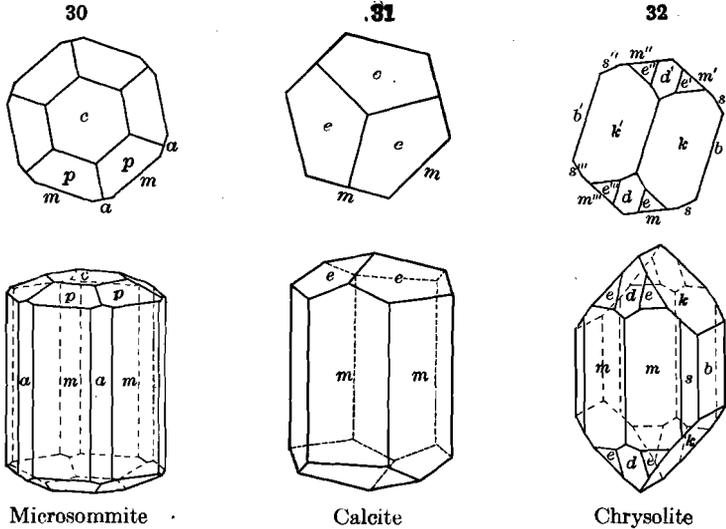
Further, there is one axis of tetragonal symmetry, a principal axis; this is the vertical crystallographic axis. There are also in a plane normal to this four axes of binary symmetry — like two and two — those of each pair at right angles to each other. Fig. 28 shows a typical tetragonal crystal, and Fig. 29 a basal projection of it, that is, a projection on the principal plane of symmetry normal to the vertical axis. See also Fig. 36 and Figs. 170–192.

* Two planes of symmetry are said to be *like* when they divide the ideal crystal into halves which are identical to each other; otherwise, they are said to be *unlike*. Axes of symmetry are also like or unlike. If a plane of symmetry includes two of the crystallographic axes, it is called an *axial plane* of symmetry. If the plane includes two or more like axes of symmetry, it is called a *principal plane* of symmetry; also an axis of symmetry in which two or more like planes of symmetry meet is a *principal axis* of symmetry.

† In describing the symmetry of the different classes, here and later, the center of symmetry is ordinarily not mentioned when its presence or absence is obvious.

III. HEXAGONAL SYSTEM. In the *Hexagonal Division* there are four axial planes of symmetry; of these three are like planes meeting at angles of 60° , their intersection-line being the vertical crystallographic axis; the fourth plane (a principal plane) is at right angles to these. There are also three other diagonal planes of symmetry meeting the three of the first set in the vertical axis, and making with them angles of 30° .

Further, there is one principal axis of hexagonal symmetry; this is the vertical crystallographic axis; at right angles to it there are also six binary axes. The last are in two sets of three each. Fig. 30 shows a typical hexagonal crystal, with a basal projection of the same. See also Fig. 37 and Figs. 220-227.



Microsommitte

Calcite

Chrysolite

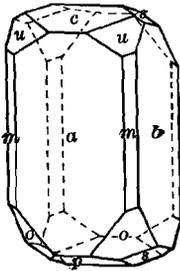
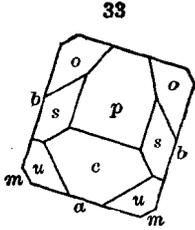
In the *Trigonal or Rhombohedral Division* of this system there are three like planes of symmetry intersecting at angles of 60° in the vertical axis. Further, the forms belonging here have a vertical principal axis of trigonal symmetry, and three horizontal axes of binary symmetry, coinciding with the horizontal crystallographic axes. Fig. 31 shows a typical rhombohedral crystal, with its basal projection. See also Figs. 243-269.

IV. ORTHORHOMBIC SYSTEM. Three unlike planes of symmetry meeting at 90° , and fixing by their intersection-lines the position of the crystallographic axes. Further, three unlike axes of binary symmetry coinciding with the last-named axes. Fig. 32 shows a typical orthorhombic crystal, with its basal projection. See also Fig. 38 and Figs. 298-320.

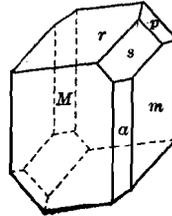
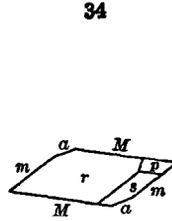
V. MONOCLINIC SYSTEM. One plane of symmetry which contains two of the crystallographic axes. Also one axis of binary symmetry, normal to this plane and coinciding with the third crystallographic axis. See Fig. 33; also Fig. 39 and Figs. 333-347.

VI. TRICLINIC SYSTEM. No plane and no axis of symmetry, but symmetry solely with respect to the central point. Figs. 34 and 40 show typical triclinic crystals. See also Figs. 359-366

26. The relations of the normal classes of the different systems are further illustrated both as regards the crystallographic axes and symmetry by the accompanying figures, 35-40. The exterior form is here that bounded by faces each of which is parallel to a plane through two of the crystallographic axes indicated by the central broken lines. Further, there is shown, within this, the combination of faces each of which joins the extremities of the unit lengths of the axes.

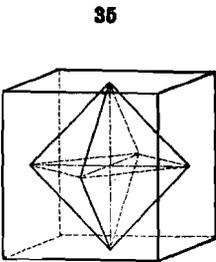


Pyroxene

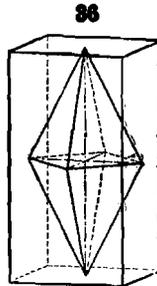


Axinite

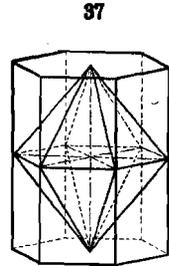
The full understanding of the subject will not be gained until after a study of the forms of each system in detail. Nevertheless the student will do well to make himself familiar at the outset with the fundamental relations here illustrated.



Isometric



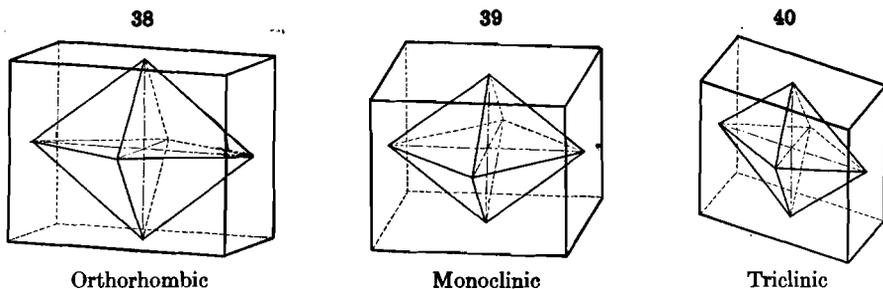
Tetragonal



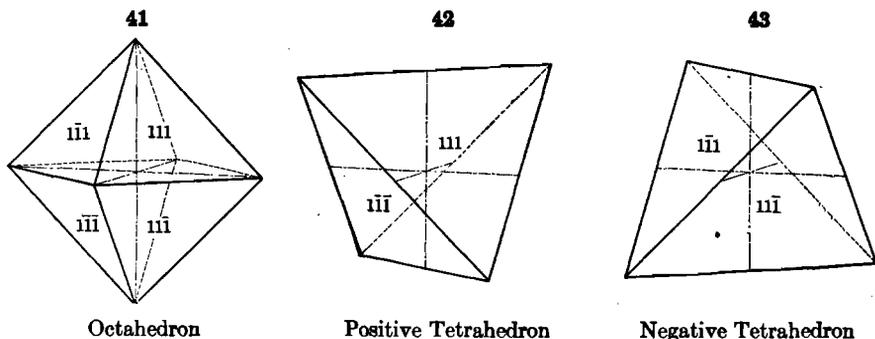
Hexagonal

It will be shown later that the symmetry of the different classes can be most clearly and easily exhibited by the use of the different projections explained in Art. 39 *et seq.*

27. **Models.** — Glass (or transparent celluloid) models illustrating the different systems, having the forms shown in Figs. 35–40, will be very useful to the student, especially in learning the fundamental relations as regards symmetry. They should show within, the crystallographic axes, and by colored threads or wires, the outlines of one or more simple forms. Models of wood are also made in great variety and perfection of form; these are indispensable to the student in mastering the principles of crystallography.



28. **So-called Holohedral and Hemihedral Forms.** — It will appear later that each crystal form * of the normal class in a given system embraces *all* the faces which have a like geometrical position with reference to the crystallographic axes; such a form is said to be *holohedral* (from ὅλος, complete, and ἔδρα, face). On the other hand, under the classes of lower symmetry, a certain form, while necessarily having all the faces which the symmetry allows, may yet have but *half* as many as the corresponding form of the normal class; these half-faced forms are sometimes called on this account *hemihedral*. Furthermore, it will be seen that, in such cases, to the given holohedral form there correspond two similar and complementary hemihedral forms, called respectively positive and negative (or right and left), which together embrace all of its faces.



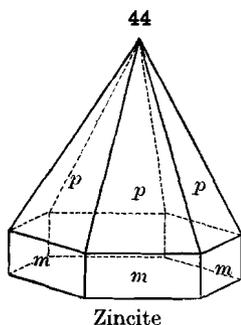
A single example will help to make the above statement intelligible. In the normal class of the isometric system, the octahedron (Fig. 41) is a "holohedral" form with all the possible faces — eight in number — which are alike in that they meet the axes at equal distances. In the tetrahedral class of the same system, the forms are referred to the same crystallographic axes, but the symmetry defined in Art. 19 (and more fully later) calls for but four similar faces having the position described. These yield a four-faced, or "hemihedral," form, the tetrahedron. Figures 42 and 43 show the positive and negative tetrahedron, which together, it will be seen, embrace all the faces of the octahedron, Fig. 41.

* The use of the word *form* is defined in Art. 37

In certain classes of still lower symmetry a given crystal form may have *ut one-quarter* of the faces belonging to the corresponding normal form, and, after the same method, such a form is sometimes called *tetartohedral*.

The development of the various possible kinds of hemihedral (and tetartohedral) forms under a given system has played a prominent part in the crystallography of the past, but it leads to much complexity and is distinctly less simple than the direct statement of the symmetry in each case. The latter method is systematically followed in this work, and the subject of hemihedrism is dismissed with the brief (and incomplete) statements of this and the following paragraphs.

29. Hemimorphic Forms. — In several of the systems, forms occur under the classes of lower symmetry than that of the normal class which are characterized by this: that the faces present are only those belonging to one extremity of an axis of symmetry (and crystallographic axis). Such forms are conveniently called *hemimorphic* (*half-form*). A simple example under the hexagonal system is given in Fig. 44. It is obvious that hemimorphic forms have no center of symmetry.



Zincite

30. Molecular Networks. — Much light has recently been thrown upon the relations existing between the different types of crystals, on the one hand, and of these to the physical properties of crystals, on the other, by the consideration of the various possible methods of grouping of the molecules of which the crystals are supposed to be built up. This subject, very early treated by Haiy and others (including J. D. Dana), was discussed at length by Frankenheim and later by

Bravais. More recently it has been extended and elaborated by Sohncke, Wulff, Schönflies, Fedorow, Barlow, and others.

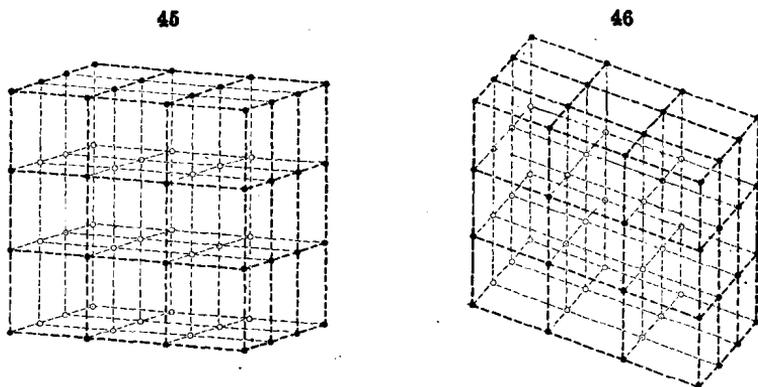
All solid bodies, as stated in Art. 7, are believed to be made up of definite physical units, called the physical, or crystal, molecules. Of the form of the molecules nothing is definitely known, and though theory has something to say about their size, it is enough here to understand that they are almost infinitely small, so small that the surface of a solid — *e.g.*, of a crystal — may appear to the touch and to the eye, even when assisted by a powerful microscope, as perfectly smooth.

The molecules are further believed to be not in contact but separated from one another — if in contact, it would be impossible to explain the motion to which the sensible heat of the body is due, or the transmission of radiation (radiant heat and light) through the mass by the wave motion of the ether, which is believed to penetrate the body.

When a body passes from the state of a liquid or a gas to that of a solid, under such conditions as to allow perfectly free action to the forces acting between the molecules, the result is a crystal of some definite type as regards symmetry. The simplest hypothesis which can be made assumes that the form of the crystal is determined by the way in which the molecules group themselves together in a position of equilibrium under the action of the intermolecular forces.

As, however, the forces between the molecules vary in magnitude and direction from one type of crystal to another, the resultant grouping of the molecules must also vary, particularly as regards the distance between them

and the angles between the planes in which they lie. This may be simply represented by a series of geometrical diagrams, showing the hypothetical groupings of points which are strictly to be regarded as the centers of gravity of the molecules themselves. Such a grouping is named a *network*, or point-system, and it is said to be *regular* when it is the same for all parallel lines and planes, however they be taken. For the fundamental observed fact, true in all simple crystals, that they have like physical properties in all parallel directions, leads to the conclusion that the grouping of the molecules must be the same about each one of them (or at least about each unit group of them), and further the same in all parallel lines and planes.



Crystal Networks

The subject may be illustrated by Figs. 45, 46 for two typical cases, which are easily understood. In Fig. 45 the most special case is represented where the points are grouped at equal distances, in planes at right angles to each other. The structure in this case obviously corresponds in symmetry to the cube described in Arts. 15 and 16, or, in other words, to the normal class of the isometric system. Again, in Fig. 46, the general case is shown where the molecules are unequally grouped in the three directions, and further these directions are oblique. The symmetry is here that of the normal class of the triclinic system.

If, in each of these cases, the figure be bounded by the simplest possible arrangement of eight points, the result is an *elementary parallelepiped*, which obviously defines the molecular structure of the whole. In the grouping of these parallelepipeds together, as described, it is obvious that in whatever direction a line be drawn through them, the points (molecules) will be spaced alike along it, and the grouping about any one of these points will be the same as about any other.

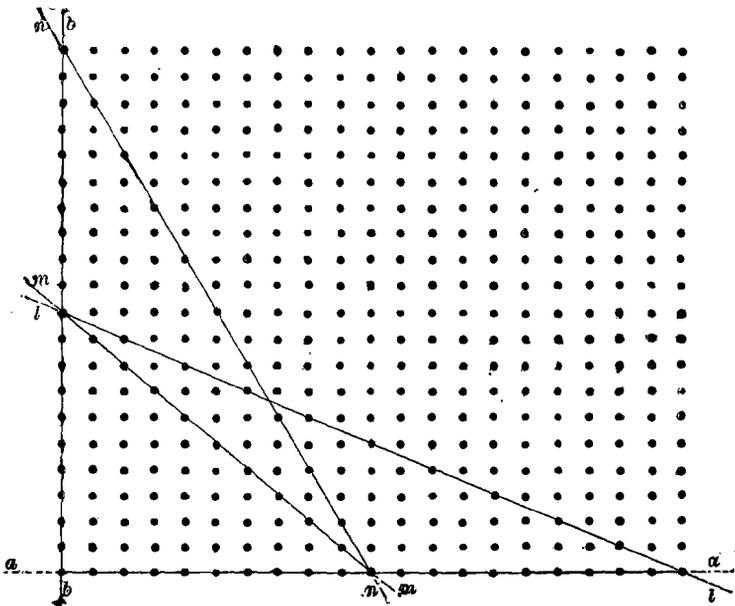
31. Certain important conclusions can be deduced from a consideration of such regular molecular networks as have been spoken of, which will be enumerated here though it is impossible to attempt a full explanation.

(1) The prominent crystalline faces must be such as include the largest number of points, that is, those in which the points are nearest together.

Thus in Fig. 47, which represents a section of a network conforming in symmetry to the structure of a normal orthorhombic crystal, the common crystalline faces would be expected to be those having the position *bb*, *aa*, *mm*,

then ll , nn , and so on. This is found to be true in the study of crystals, for the common forms are, in nearly all cases, those whose position bears some simple relation to the assumed axes; forms whose position is complex are usually present only as small faces on the simple predominating forms, that is, as modifications of them. So-called *vicinal* forms, that is, forms taking the place of the simple fundamental forms to which they approximate very closely in angular position, are exceptional.

47

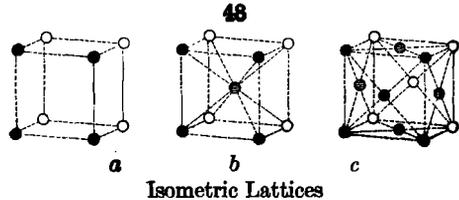


Orthorhombic Point System

(2) When a variety of faces occur on the same crystal, the numerical relation existing between them (that which fixes their position) must be rational and, as stated in (1), a simple numerical ratio is to be expected in the common cases. This, as explained later, is found by experience to be a fundamental law of all crystals. Thus, in Fig. 47, starting with a face meeting the section in mm , ll would be a common face, and for it the ratio is 1 : 2 in the directions b and a ; nn would be also common with the ratio 2 : 1.

(3) If a crystal shows the natural easy fracture, called cleavage, due to a minimum of cohesion, the cleavage surface must be a surface of relatively great molecular crowding, that is, one of the common or fundamental faces. This follows (and thus gives a partial, though not complete, explanation of cleavage) since it admits of easy proof that that plane in which the points are closest together is farthest separated from the next molecular plane. Thus in Fig. 47 compare the distance separating two adjoining planes parallel to bb or aa ; then two parallel to mm , ll , nn , etc. Illustrations of the above will be found under the special discussion of the subject of cleavage.

32. Kinds of Molecular Groupings. — The discussion on the basis just described shows that there are fourteen possible types of arrangement of the molecules. These agree as to their symmetry with the seven classes defined in Art. 25 as representing respectively the normal classes of the six systems with also that of the trigonal (or the rhombohedral) division of the hexagonal system. Of the fourteen, three groupings belong to the isometric system (these are shown, for sake of illustration, in Fig. 48 from Groth; *a*, cube lattice; *b*, cube-centered lattice; *c*, face centered cube lattice); two to the tetragonal; one each to the hexagonal and the rhombohedral; four to the orthorhombic system; two to the monoclinic, and one to the triclinic.



In its simplest form, as above outlined, the theory fails to explain the existence of the classes under the several systems of a symmetry lower than that of the normal class. It has been shown, however, by Sohncke and later by Fedorow, Schönflies and Barlow, that the theory admits of extension. The idea supposed by Sohncke is this: that, instead of the simple form shown, the network may consist of a double system, one of which may be conceived of as having a position relative to the other (1) as if pushed to one side, or (2) as if rotated about an axis, or finally (3) as if both rotated as in (2) and displaced as in (1). The complexity of the subject makes it impossible to develop it here. It must suffice to say that with this extension Sohncke concludes that there are 65 possible groups. This number has been further extended to 230 by the other authors named, but it still remains true that these fall into 32 distinct types as regards symmetry, and thus all the observed groups of forms among crystals, described under the several systems, have a theoretical explanation.

Literature. — A complete understanding of this subject can only be gained by a careful study of the many papers devoted to it. An excellent and very clear summary of the whole subject is given by Groth in the fourth edition of his *Physikalische Krystallographie*, 1905, and by Sommerfeldt, in his *Physikalische Kristallographie*, 1907.

33. X-Rays and Crystal Structure. — In 1912, while attempting to prove a similarity in character between X-rays and light, Dr. Laue, of the University of Zurich conceived the idea of using the ordered arrangement of the molecules or atoms of a crystal as a "diffraction grating" for their analysis. By placing a photographic plate behind a crystal section which in turn lay in the path of a beam of X-rays he found that not only did the developed plate show a dark spot in its center where the direct pencil of the X-rays had hit it but it also showed a large number of smaller spots arranged around the center in a regular geometrical pattern. This pattern was formed by the interference of waves which had been diffracted in different directions by the molecular structure of the crystal. In this way he succeeded in proving that X-rays belong to the same class of phenomena as light but with a much shorter wave length. The experiment showed indeed that the wave lengths of the X-rays must be comparable to the distances between the layers of molecular particles of crystals. Another, and, from the crystallographic point of view, a very important, result of this investigation was the furnishing of a

method for the study of the internal structure of crystals. The position of the smaller dark spots in the Laue photographs corresponded to that of various planes existing in the crystal network parallel to possible crystal faces and their arrangement indicated the symmetry of the crystal.

Following these investigations of Laue and his colleagues another fruitful method of investigation of crystal structure by means of X-rays was devised by W. H. and W. L. Bragg. In this method the beam of X-rays meets the crystal section with varying acute angles of incidence and the reflection of the rays is studied. The X-rays are not reflected from the surface of the section like light rays but because of their short wave lengths penetrate the crystal section and are reflected from the successive layers of its molecular structure. In studying the reflection phenomena we have to consider the effect upon each other of these different wave trains originating from the different layers of the crystal. In general these various reflected waves would be in different phases of vibration and so would tend to interfere with each other with the consequent cessation of all vibrations. But with a certain angle of incidence and reflection it would happen that the different reflected rays would possess on emergence from the crystal the same phase of vibration and would therefore reinforce each other. This angle would vary with the wave length of the X-ray used (for it has been found that the wave length of X-rays varies with the metal that is used as the anticathode in the X-ray bulb) and with the spacing between the molecular layers of the mineral used. It is also obvious that there might be other angles of incidence at which the successive wave trains would each differ in phase by two or even more whole wave lengths from the preceding one and a similar strong reflected beam obtained. By the use of a special X-ray spectrometer the angles at which these reflections take place can be accurately measured. If the character of the X-ray used is therefore kept constant these angles of reflection give the data necessary for calculating the distance between the successive molecular layers in the particular mineral used and for the direction perpendicular to the surface used for reflection. The spacing of the molecular layers was found to vary with different substances and in different directions in the same substance and by making a series of observations it has been possible to arrive at some very interesting conclusions as to the character of the molecular structure of certain minerals as well as to the relationship existing between the structures of different but related compounds. The possibilities lying in these methods of attack are very great and unquestionably much new information concerning crystal structure will soon be available. An excellent summary of the methods employed and the results already obtained will be found in "X-rays and Crystal Structure" by W. H. and W. L. Bragg, 1915.

GENERAL MATHEMATICAL RELATIONS OF CRYSTALS

34. Axial Ratio, Axial Plane. — The crystallographic axes have been defined (Art. 22) as certain lines, usually determined by the symmetry, which are used in the description of the faces of crystals, and in the determination of their position and angular inclination. With these objects in view, certain