

## Acknowledgments

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# Graphical Determination of Circulating Loads in Crushing Circuits

M.R. Moharam

**Abstract**—The well-known formulae of screening efficiency and circulating load calculations in closed crushing circuits were exploited to achieve graphical estimation of these parameters. Three basic charts were proposed and constructed. The first one is for the regular closed circuits where the new feed goes to the crusher. The second is for the reverse closed circuits in which the new feed reported to the screen is free of screen undersize particles. The third chart controls the reverse circuits when they are receiving new incoming feeds containing undersize material. Compound circuits were also reanalyzed and a concept of equivalent simple circuit was introduced to enable graphical manipulation of these complex circuits with the help of the aforementioned charts.

## Introduction

The usual arrangement in size reduction plants provides that a comminuting machine works in closed circuit with a sizing and/or classifying device. The major advantage in utilizing these closed circuits is to reduce in size the largest possible tonnage to a given specification using the least amount of energy and creating the smallest possible percentage of extreme fines.

In selecting size reduction equipment, preparing flowsheets, and designing plants, there is a need for calculating the circulating load of closed circuits. For this purpose, simple and accurate equations are very useful. Though these equations are well known in the earlier literature (Davis, 1925; Taggart, 1945) they still receive attention, investigation, and—to some extent—modification in new publications (Flavel, 1977; Karra, 1979; and Hukki, 1979).

Being a key parameter in comminution closed circuits, performance of sizing devices, as expressed by separation efficiency, should always be analyzed in combination with the respective circulating load values. In this present work, the classical definition of screening efficiency, which was adopted by the Vibrating Screen Manufacturers Association (Anon., 1967) will be considered. This definition states that screen efficiency is the

percent ratio between the actual amount of screen undersize and the theoretical amount of undersize material in screen feed.

Well-known expressions will be considered throughout this work, but to develop and construct new set of charts enabling graphic estimation of the two essential parameters in crushing-sizing closed circuits, i.e., screening efficiency and percent circulating load will be used. In addition, a modification in the analytical processing of the compound circuits will be introduced in order to adopt these circuits for graphical solutions.

## Regular Closed Circuits

Fig. 1 shows the general arrangement of this type of circuit, in which the new feed goes to the crusher. After the complete buildup of the circulating load, the circuit reaches equilibrium, and the new tonnage of feed  $F$  equals the screen undersize product in tons  $P_s$ .

From the basic material balance and by recalling the aforementioned definition of screening efficiency, the circulating load  $R_g$ , defined as percent of new feed, could be expressed as:

$$R_g = \frac{10^6}{E p_c} - 100 \quad (1)$$

where  $p_c$  is the percentage undersize in the crusher product and  $E$  is the screen efficiency.

For a specific value of screen efficiency  $E$ , Eq. 1 represents a straight line relationship between the two variables  $R_g$  and  $1/p_c$ . This straight line is identified by the slope  $(10^6/E)$  and by intercepting the ordinate at  $R_g = -100$ . Consequently, for the different values of  $E$ , a set of straight lines with the aforemen-

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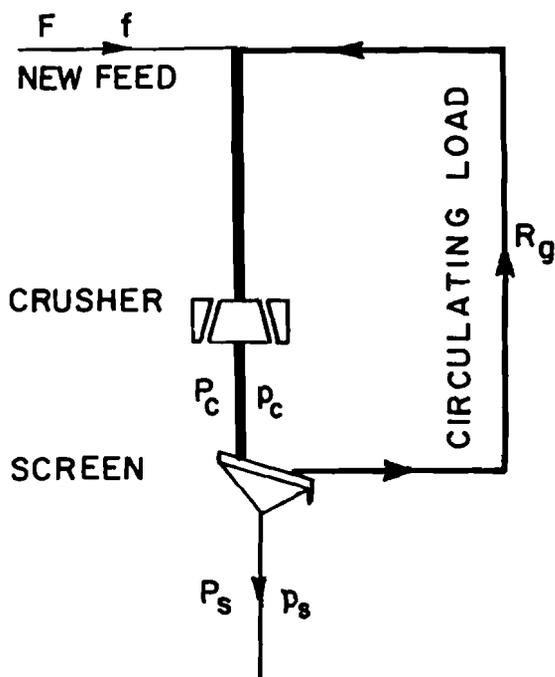


Fig. 1—General arrangement of regular closed circuit.

tioned specified parameters are drawn to construct Fig. 4. Each of these lines represents the dependence of the circulating load on the reciprocal value of the percent undersize material in the crusher product if the stated value of screening efficiency is maintained. A conversion scale from  $1/p_c$  to  $p_c$  is introduced at the bottom of the chart.

In order to use Fig. 4 to estimate efficiency, a scale for the values of the screen undersize product expressed as percent of screen feed ( $p_s$ ) was established at the left hand side of the chart. These values were calculated by redefining screen efficiency as:

$$E = \frac{p_s}{p_c} \times 100 \quad (2)$$

which can be easily rearranged into:

$$\frac{p_c}{p_s} \times 10^4 = \frac{10^6}{E} \quad (3)$$

In Eq. 3, the right-hand side is equivalent to the slope of any efficiency line as it was expressed by Eq. 1. Also, when substituting a value of  $E = 100\%$  in Eq. 3, identical numerical values for both ( $p_c$ ) and ( $p_s$ ) are capable to fix the slope value of the 100% efficiency line. Consequently, a set of ( $p_s$ ) values were marked on the scale by drawing vertical lines from the respective values of  $p_c$  to intersect the 100% efficiency line, and then horizontal lines intersecting the  $P_s$  scale.

The example shown in Fig. 4 indicates 65% efficiency and 180% circulating load when  $p_c = 55\%$  and  $p_s = 35.5\%$ .

### Reverse Closed Circuits

In this type of circuit the new feed reports to the sizing device. By referring to Fig. 2 and applying the basic principles of material balance at equilibrium state conditions, one can reach the following:

$$R_v = \frac{1}{p_c} \left( \frac{10^6}{E} - 100f \right) \quad (4)$$

where  $R_v$  is the circulating load (in reverse circuit) expressed as percent of new feed,  $f$  is the percentage screen undersize material in the new income feed ( $F$  tons), and all other symbols have the same nomenclatures as mentioned before.

If the new feed is free of screen undersize material, i.e.,  $f = 0$ , the circulating load ( $R_{v0}$ ) will be:

$$R_{v0} = \frac{1}{p_c} \cdot \frac{10^6}{E} \quad (5)$$

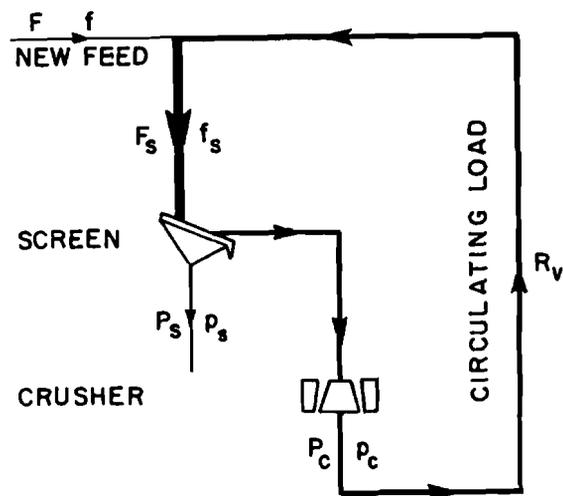


Fig. 2—General arrangement of reverse closed circuit.

Chart for Reverse Closed Circuit with New Feed Free of Screen Undersize Material. Eq. 5 shows that a straight line passing through the origin and having a slope of  $(10^6/E)$  represents the relationship between the two variables  $R_{v0}$  and  $1/p_c$ . For the different recommended values of  $E$  a group of these lines is available, as shown on Fig. 5. A conversion scale to convert  $1/p_c$  to  $p_c$  is shown at the bottom of the chart.

In order to develop this chart for efficiency estimation, two other scales should be established: One for the screen undersize product expressed as percent of screen feed  $p_s$ , and the second for percent undersize in screen feed  $f_s$ . Such requirement is attributed to the basic definition of screening efficiency in reverse closed circuit as:

$$E = \frac{p_s}{f_s} \times 100 \quad (6)$$

Identified in terms of circulating load,  $f_s$  and  $p_s$  will be:

$$f_s = p_c \frac{R_{v0}}{R_{v0} + 100} \quad (7)$$

$$\frac{f_s}{p_s} \times 10^4 = \frac{10^6}{E} = R_{v0} \cdot p_c \quad (8)$$

and

$$p_s = \frac{10^4}{R_{v0} + 100} \quad (9)$$

Eq. 9 was solved for arbitrary values of  $p_s$  and the corresponding values of  $R_{v0}$  were estimated. Then a scale for  $p_s$  was established at the left hand side of the chart opposite to  $R_{v0}$  scale but shifted upward by 100% of circulating load scale. This conclusion could be approved by comparing Eq. 5 with Eq. 1 to notice that:

$$R_{v0} = R_v + 100\% \quad (10)$$

In order to establish a scale for the variable  $f_s$ , Eq. 8 was exploited and  $E$  was substituted by 100%. The  $f_s$  scale was placed at the right hand top side of the chart.

The dashed lines on the chart show 235%  $R_{v0}$  and 67%  $E$  if  $p_c = 64\%$ ,  $p_s = 30\%$ , and consequently  $f_s = 45\%$ .

Chart for Reverse Closed Circuit When New Feed Contains Screen Undersize Material. Basically, the chart in Fig. 6 is similar to the chart in Fig. 5, but a slope reduction scale was established at the right-hand side.

This scale does the required compensation for the term  $(100f)$  involved in the essential Eq. 4. The values marked on this scale are the corresponding slopes of the different efficiency lines. The nature of such scale is easily understood if one notices that the relationship between the circulating load and  $1/p_c$  has slopes of  $10^6/E$  in the reverse circuits of  $f = 0$ , while it has slope values of  $(10^6/E - 100f)$  in the reverse circuits of  $f \neq 0$ .

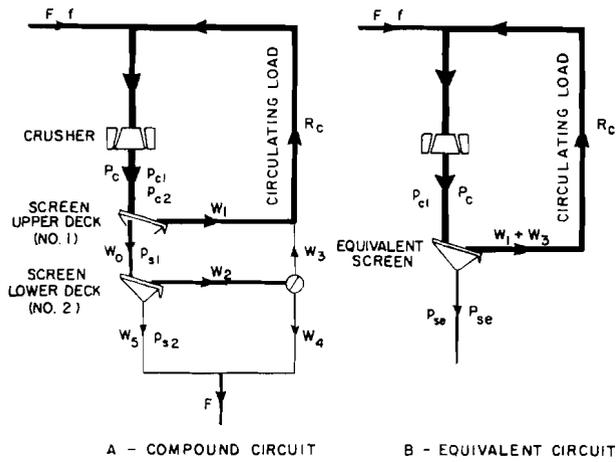


Fig. 3—Substitution of a compound closed circuit with a simple equivalent one.

The dashed lines and arrows drawn on the chart indicate 245% circulating load for a reverse closed circuit that has the operating conditions of:  $E = 65\%$ ,  $p_c = 55\%$ , and  $f = 20\%$ .

### Impracticable Zones on the Charts

Theoretically speaking, the following conditions must be fulfilled in order to achieve correct application of all the aforementioned formulae and charts:  $E \leq 100\%$ ,  $p_c \leq 100\%$ ,  $1/p_c \geq 0.01$ ; and from Eq. 10:  $R_{v0} \geq 100\%$ . Consequently, in the proposed charts, the areas in which values beyond these extremes may exist are considered and marked "impracticable zones."

### Complex Circuits and the Concept of Simple Equivalent Circuit

In crushing plants, it is not unusual to find closed circuits incorporating more than one screen. Fig. 3-A shows the general schematic arrangement of one of these compound circuits. In such a circuit, part of the oversize material from the bottom screen is returned to the crusher for further crushing when it is intended to adjust product size to meet a specified technological requirement. In this case the circulating load is compounded of top screen oversize product and that portion recycled from the lower screen oversize material.

If it is possible to transform a complex multiscreen circuit into a simple equivalent comprising one screen only, the charts will

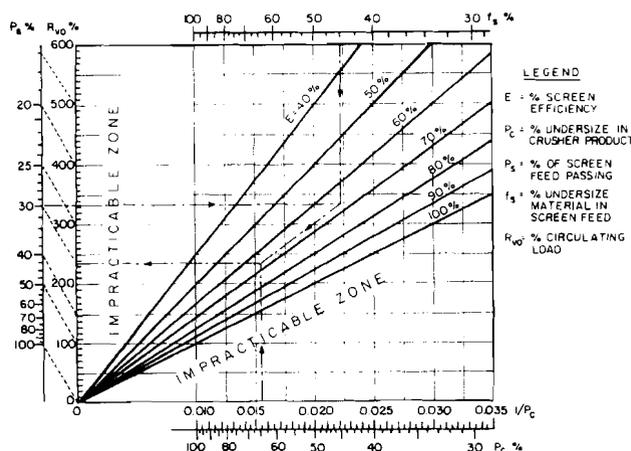


Fig. 5— Chart for determining circulating load in reverse closed circuit. New incoming feed is free of undersize material.

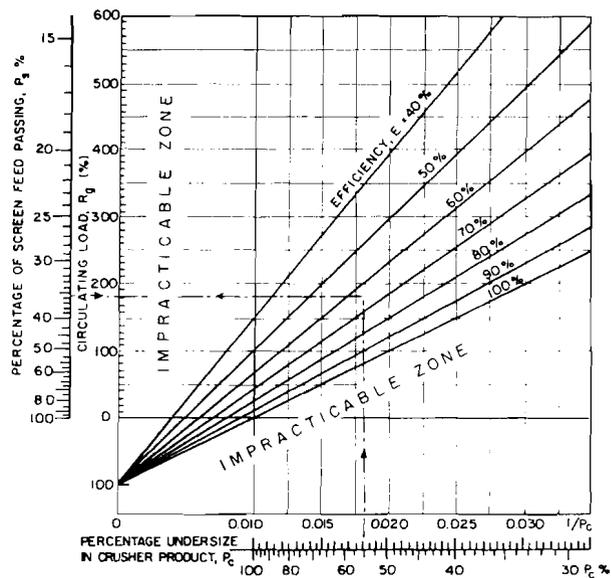


Fig. 4— Chart for determination of circulating load in regular closed circuit.

be valid for use with this equivalent circuit and consequently with the original complex one.

Fig. 3-B represents the simple circuit which is assumed to be equivalent to the compound one shown in Fig. 3-A. Tonnage for each branch of the two circuits is identified by the adjoining  $W$ . These weights are self-explanatory on the figures.

In the equivalent circuit, the top screen of the original complex circuit is assumed to retain the tonnage ( $W_1 + W_2$ ) as oversize.  $W_0$  is also assumed to be completely undersize material. Consequently the efficiency of that screen will change from  $E_1$  to another smaller value (equivalent efficiency  $E_e$ ), and the screen itself will be redefined as equivalent screen. Tons of undersize passing this equivalent screen are  $P_{se}$ . In Fig. 3,  $p_{s1}$ ,  $p_{s2}$ , and  $p_{se}$  are the undersize products expressed as percent of screen feeds for top, bottom, and equivalent screen respectively; and  $p_{c1}$  and  $p_{c2}$  are the percent undersize in crusher discharge for top and bottom screens, respectively.

Efficiency of the equivalent screen can be derived by following the succeeding analysis.

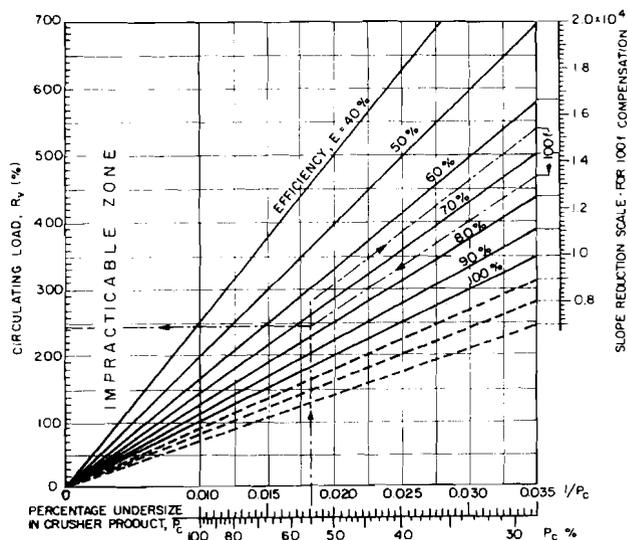


Fig. 6— Chart for estimation of circulating load in reverse closed circuit. New incoming feed contains undersize material.

From Fig. 3-A:

$$E_1 = \frac{p_{s1}}{p_{c1}} \times 100 = \frac{10^4 W_0}{p_c \cdot p_{c1}} \quad (11)$$

and

$$E_2 = \frac{p_{s2}}{p_{c2}} \times 100 = \frac{10^4 W_5}{p_c \cdot p_{c2}} \quad (12)$$

Similarly, from Fig. 3-B:

$$E_e = \frac{p_{se}}{p_{c1}} \times 100 = \frac{10^4 (W_0 - W_2)}{p_c \cdot p_{c1}} \quad (13)$$

By rearrangement and substitution:

$$E_e = \frac{W_0 - k(W_0 - W_5)}{p_{c1} \cdot p_c} \times 10^4 \quad (14)$$

$$= E_1(1 - k) - kE_2 \frac{p_{c2}}{p_{c1}}$$

where  $k$  is the fraction of the second (bottom) screen oversize returned to the crusher.

The two quantities  $E_1$  and  $E_2$  also can be evaluated by the help of Fig. 4.  $E_e$  is calculated by substituting in Eq. 14.

Following the same procedures mentioned for the regular closed circuit,  $E_e$  is combined with  $p_{c1}$  to estimate graphically the circulating load in the complex circuit  $R_c$ .

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# Colorado Nahcolite Deposits: Geology and Outlook for Development

John R. Dyni

**Abstract**—Colorado nahcolite deposits, estimated at 29 Gt, formed in an asymmetric permanent-lake basin whose waters were rich in algae and sulfate-reducing bacteria. Nahcolite ( $\text{NaHCO}_3$ ) occurs in oil shale of the Green River Formation (Eocene) as aggregates (62%), disseminated crystals (24%), and impure beds (13%). Because much of the nahcolite is nonbedded, it will be mined with oil shale and can be concentrated to marketable grade by crushing and screening. Impurities in nahcolite are kerogen, quartz, pyrite, and carbonate minerals.

A promising new market for nahcolite is in the process of removal of sulfur dioxide from industrial stack gases. When heated, nahcolite reacts with sulfur dioxide to form sodium sulfate. Differences in thermally induced microporosity may explain why nahcolite is more reactive than trona, a possible competitor with nahcolite for  $\text{SO}_2$  removal. Limited production of nahcolite at one or possibly two localities is expected in the next several years, but large-scale production is probably at least 10 years away.

## Introduction

Enormous deposits of nahcolite ( $\text{NaHCO}_3$ ) are associated with rich, thick oil shales of the lacustrine Green River Formation (Eocene) in northwestern Colorado. After the Wyoming trona deposits, Colorado's nahcolite deposits constitute the second largest known resource of sodium carbonate in the world, estimated at 26-29 Gt (Beard, Tait, and Smith, 1974; Dyni, 1974). Although this resource is confined to a deeply buried zone in the northern part of the Piceance Creek Basin, it seems likely in view of current energy needs that some nahcolite will be coproduced with oil shale within the next 5-10 years. Nahcolite is now being mined from part of the deposits for experimental purposes.

## Geology

Evaporite deposits of nahcolite and halite occur in dolomitic oil shale of the Parachute Creek Member of the Green River Formation in the northern part of the Piceance Creek Basin, Rio Blanco County, CO. Nahcolitic oil shale more than 30 m thick underlies about 660 km<sup>2</sup> and reaches a maximum thickness of about 310 m at the basin depocenter (Fig. 1). The depth from the surface to the top of the nahcolite deposits ranges from 425-575 m.

The stratigraphy of the nahcolite deposits and enclosing beds between basin center and the outcrop at the north end of the basin is illustrated in Fig. 2. The upper limit of the principal nahcolite deposits is a dissolution surface. The oil shales above the dissolution surface, extending stratigraphically upward into the middle part of the Mahogany oil-shale zone, are much fractured and contain many solution breccias, vugs, and crystal cavities. These solution features indicate that the evaporite deposits were originally much thicker, perhaps by a third of that preserved below the dissolution surface. The zone of leached and broken rock is about 120-200 m thick and forms an aquifer containing saline water in contact with the nahcolite rocks below. The L-5 lean oil-shale zone, in particular, contains many solution breccias and solution cavities and probably includes the least competent rocks in the leached zone. At the north end of the basin, the saline facies thins abruptly and lenses out into kerogenaceous analcimic and illitic shales within a distance of less than 4-5 km. On the outcrop at the mouth of Piceance

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