

## A probabilistic approach to column flotation

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### ABSTRACT

Froth flotation has been employed as a successful means of concentration of metalliferous ores. Column flotation is a relatively new development. The process involves selective attachment of particles of certain minerals out of a slurry, descending a vertical column, to ascending air bubbles. There are some particles which attach themselves to the ascending air bubbles from the collection zone and go right up to the cleaning zone whereas others are attached for a while and are, then, detached. For certain other particles, the process of attachment and detachment is repeated. The probabilities of change from the attachment to detachment state of a particle are determined from the recoveries of the two zones. The paper explains the steady state probabilities of a particle from the probabilities of change from one state to another based on the principle of Markov process. © 2004 SDU. All rights reserved.

Keywords: Column flotation; Collection zone; Cleaning zone

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### 1. INTRODUCTION

The effectiveness of a flotation process in separation of valuable minerals from the gangue minerals depends on the particles of one type being hydrophobic and the other, hydrophilic. These properties may be natural or enhanced by the action of certain reagents such as collectors and depressants. Collectors are added to a pulp to make some particles differentially hydrophobic while depressants make the other particles differentially more susceptible to be wetted. However, prior to the application of any mineral separation process, the particles of different minerals should be liberated by comminution. Additionally, for the flotation process to be effective, the particles should be fine enough to be lifted by the air bubbles but not so fine as to make multi-mineral coagulates.

In column flotation a tall vessel, instead of a conventional flotation cell, is employed. Appropriate reagents such as frothers, collectors and depressants are added and the conditioned feed slurry is fed into the tall vessel where it descends against the counter-currently rising air bubbles generated by a sparger at the bottom of the column as shown in Figure 1.

At the interface of the pulp and froth, the movement of the particles is in both directions. Particles are collected and transported to the cleaning zone (froth zone) when attached to the air bubbles. Some of them are dropped back from the cleaning zone to the collection zone (pulp zone) due mainly to coalescence of the air bubbles. Some of the particles, which drop out of the cleaning zone, may again be collected.

As a particle is introduced into the flotation columns, it either attaches itself to an air bubble or remains in the pulp. However, the particle which is initially attached to an air bubble may possibly be subsequently detached and perhaps reattached again. The probabilities of the change from one state to the other can be determined from the recoveries of the two zones. The steady state probabilities are, in turn, determined from the probabilities of the change of state.

### 2. THE APPROACH

Several investigators have made measurements of the collection zone recoveries ( $R_c$ ) and cleaning zone recoveries ( $R_f$ ). Contini *et al.* (1988) have described an experimental method for the measurement of both the rate constant in the collection zone and froth zone recovery. It acquires recovery data from two modes of operation: counter-current and co-current. The measured recoveries are fitted to the equations that

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express flotation in the two modes to yield rate constants. Falutsu and Dobby (1989) developed a method which measures the froth drop back and the collection zone recovery by modifying the conventional laboratory column. Yu and Finch (1990) have reported a method of measuring individual zone performance by a series of experiments at different ratios of froth length and total length of the column.

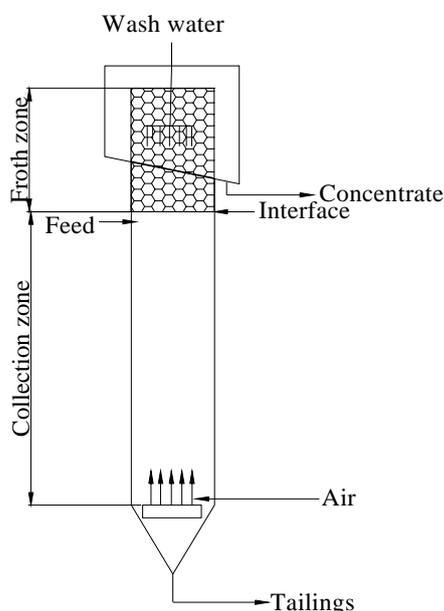


Figure 1. Zones of column flotation

For the purpose of this paper, the method developed by Falutsu and Dobby (1989) was employed. It allows the direct measurement of froth drop back,  $D$  and the collection zone recovery.

The measured values enable the determination of the probabilities of change. The method restricts the interaction between the two zones by isolating the cleaning zone from the collection zone. Moreover, the flotation tailings, concentrates and froth drop backs are discharged through three separate outlets.

The mathematical statement is as follows:

$$R_f = C/(C+D), \quad R_c = (C+D)/(C+D+T), \quad R = R_f \times R_c = C/(C+D+T)$$

Where :

$C$  = Mass of solids reporting to concentrate

$D$  = Mass of solids reporting to drop back section

$T$  = Mass of solids reporting to tailings

$R_f$  = Cleaning zone recovery

$R_c$  = Collection zone recovery and

$R$  = Total recovery from flotation of a pure single mineral.

The individual particles inside the flotation column are, either attached to the air bubbles or detached from the air bubbles. This situation can be modelled by separate states describing a particle either in attachment or detachment based on the principle of Markov process (Ullmann, 1976). Let the state of attachment to an air bubble be called state 1 and that of detachment as state 2. Figure 2 represents a probability network diagram. The node describes the state of attachment or detachment. The upward branches indicate moving to state 1 and downward branches to state 2.

### 3. PROBABILITY OF STATE CHANGE

It may be seen from Figure 2 that if a particle is in state 1 and continues to remain in the same state during the succeeding times, then it is well attached to an air bubble and is more likely to be recovered in the concentrate. Therefore, the probability of attachment will be more likely to be equal to the recovery  $C/(C+D+T)$ . The probability of it switching to state 2 is, therefore,  $1 - C/(C+D+T)$  or  $(D+T)/(C+D+T)$  which means that if a particle is detached; it will be collected either in the tailings or the drop back section. Similarly, if a particle is in state 2 and continues to remain in state 2 during the succeeding times, it remains detached

and is more likely to be recovered in the tailings. Therefore, the probability of detachment will be more likely equal to the tailings recovery,  $T/(C+D+T)$ . The probability of it switching to state 1 is  $1-T/(C+D+T)$  or  $(C+D)/(C+D+T)$  which is equal to the collection zone recovery. Table 1 shows the probability of state change.

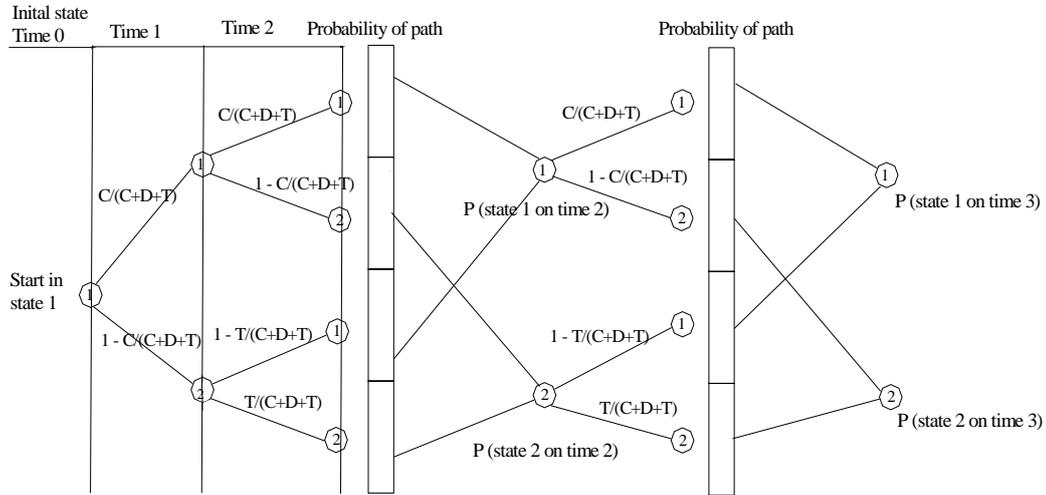


Figure 2. Possible paths of a particle in column flotation

Table 1  
 Probabilities of change

To From	Attachment (State 1)	Detachment (State 2)
Attachment (State 1)	$C/(C+D+T)$	$(D+T)/(C+D+T)$
Detachment (State 2)	$(C+D)/(C+D+T)$	$T/(C+D+T)$

From Figure 2 the results may be drawn as shown in Table 2.

Table 2  
 Probability of the paths of particles

Path No.	Time 0	Time 1	Time 2	Probability of Path
1	State 1	State 1	State 1	$[C/(C+D+T)][C/(C+D+T)]$
2	State 1	State 1	State 2	$[C/(C+D+T)][1-C/(C+D+T)]$
3	State 1	State 2	State 1	$[1-C/(C+D+T)][1-T/(C+D+T)]$
4	State 1	State 2	State 2	$[1-C/(C+D+T)][T/(C+D+T)]$

$P(\text{State 1 on time 2}/\text{State 1 on time 0})$  = probability that the particle will be in state 1 at the end of time 2 given that it was in state 1 at time 0 = probability of path No. 1 and probability of path No. 3  
 $= [C/(C+D+T)][C/(C+D+T)] + [1-C/(C+D+T)][1-T/(C+D+T)]$

$P(\text{State 2 on time 2}/\text{State 1 on time 0})$  = probability of the path No. 2 and probability of path No. 4  
 $= [C/(C+D+T)][1-C/(C+D+T)] + [1-C/(C+D+T)][T/(C+D+T)]$

Similarly at time 3, the probability will be :

$P(\text{state 1 on time 3}/\text{State 1 on time 0}) = C/(C+D+T) P(\text{State 1 on time 2}/\text{State 1 on time 0}) + (1 - T/(C+D+T)) P(\text{state 2 on time 2}/\text{State 1 on time 0})$  (1)

$= (C/(C+D+T))[C/(C+D+T)] (C/(C+D+T) + (1 - C/(C+D+T)) (1 - T/(C+D+T))] + ((1 - T/(C+D+T))[C/(C+D+T)] (1 - C/(C+D+T)) + (1 - C/(C+D+T))(T/(C+D+T))]$

$P(\text{State 2 on time 3}/\text{State 1 on time 0}) = 1 - P(\text{State 1 on time 3}/\text{State 1 on time 0})$  (2)

The procedure is continued in order to calculate the probability of the particle being in state 1 at any future time (n) given that it was in state 1 at time 0. The same procedure will be used for the calculation of the probability of the particle being in state 1 at any future time (n) given that it was in state 2 at time 0.

In view of the pattern of Equations (1) and (2), four general equations relating "the state probabilities" can be then written as follows:

$P(\text{State 1 on time n}/\text{State 1 on time 0}) = (C/(C+D+T)) P(\text{State 1 on time n-1}/\text{State 1 on time 0}) + (1 - T/(C+D+T)) P(\text{State 2 on time n-1}/\text{State 1 on time 0})$  (3)

$$P(\text{State 2 on time } n/\text{State 1 on time } 0) = 1 - P(\text{State 1 on time } n/\text{State 1 on time } 0) \quad (4)$$

$$P(\text{State 1 on time } n/\text{State 2 on time } 0) = (C/(C+D+T)) P(\text{State 1 on time } n-1/\text{State 2 on time } 0) + (1 - T/(C+D+T)) P(\text{State 2 on time } n-1/\text{State 2 on time } 0) \quad (5)$$

$$P(\text{State 2 on time } n/\text{State 2 on time } 0) = 1 - P(\text{State 1 on time } n/\text{State 2 on time } 0) \quad (6)$$

The probability at which "the state probabilities" for time  $n$  and for time  $n-1$  are nearly identical or the same then it is called "the steady state probability of being in state 1 ( $P_1$ )". The corresponding steady state probability of being in state 2 is equal to  $(1 - P_1)$ .

#### 4. CONCLUSIONS

In column flotation, there exists a relationship between the probability of particles being attached to or detached from air bubbles, to the corresponding values of the recoveries.

The probability of a particle at the state of attachment to an air bubble at any future time ( $n$ ), given that it was in the same state at the starting time, is equal to the total recovery multiplied by probability of the particle being in the state of attachment at the previous time ( $n-1$ ), given that it was attached in starting time plus collection zone recovery multiplied by the probability of the particle being at the state of detachment at the previous time given that it was attached at the starting time.

The probability of a particle at the state of attachment to an air bubble at any future time ( $n$ ), given that it was in the state of detachment at the starting time, is equal to the total recovery multiplied by probability of the particle being in the state of attachment at the previous time ( $n-1$ ), given that it was detached in starting time plus collection zone recovery multiplied by the probability of the particle being at the state of detachment at the previous time, given that it was detached at the starting time.

The probability of a particle at the state of detachment at any future time ( $n$ ), given that it was in the state of attachment at the starting time, is equal to one minus the probability of the particle at the state of attachment to an air bubble at any future time ( $n$ ), given that it was in the same state at the starting time. A similar relationship exists for the probability of a particle at the state of detachment at any future time ( $n$ ), given that it was in the state of detachment at the starting time. The steady state probabilities are attained when the state probabilities for time ( $n$ ) and time ( $n - 1$ ) are identical or the same.

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